Convergence update and New Curvature metric

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Goal is to determine if the search space is "locally convex"

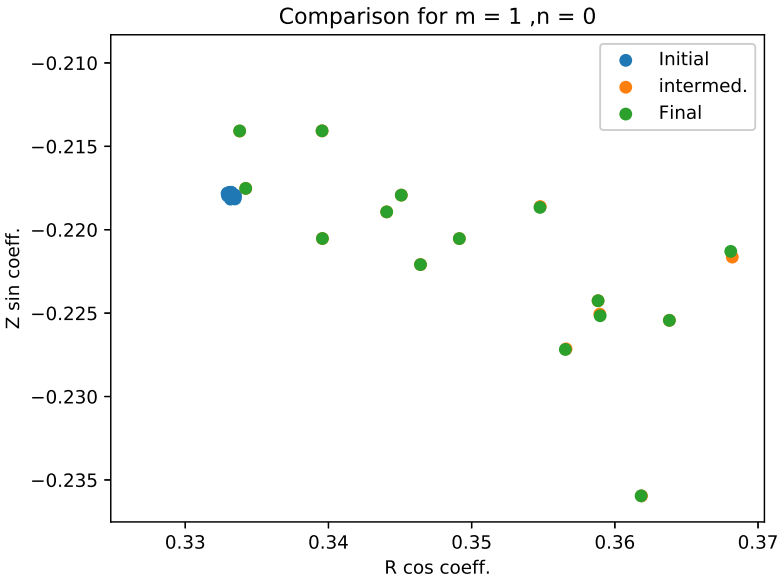
A set, $S \in \mathbb{R}^n$ is convex, if for any points, $x_1, x_2 \in S$, the straight line joining x_1, x_2 lies entirely in S
A function f is convex if its domain, S is a convex set, and:

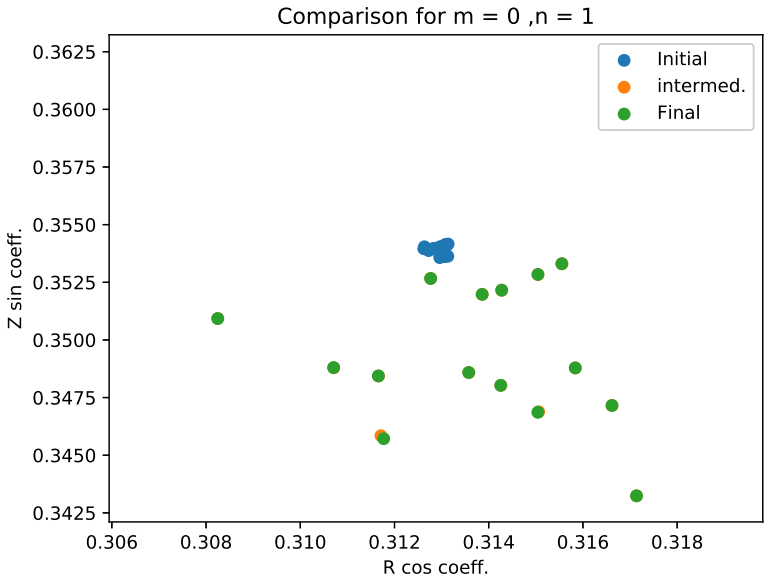
$$f\left(\alpha x + (1-\alpha)\, y\right) \leq \alpha f(x) + (1-\alpha)\, f(y)\, \forall\, \alpha \in [0,1]$$

f is locally convex if it is convex for some region of space
When considering solutions to convex functions of the form $f : \mathbb{R}^n \to \mathbb{R}$, if f is locally convex then a local solution is a global for that region of space. This means algorithms based on following descent paths will yield global solutions.

Question: Is our space locally convex?
Answer: Last week indicated that it was not convex

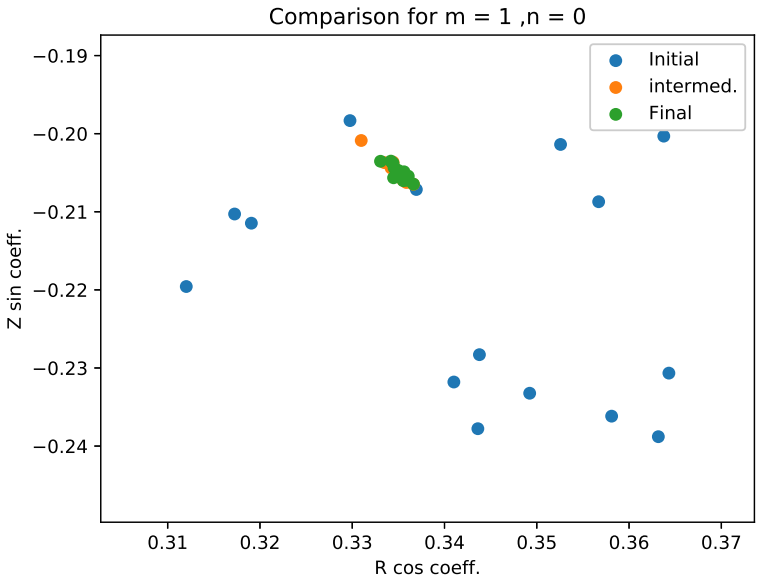
Suggestion 1: Reduce variation

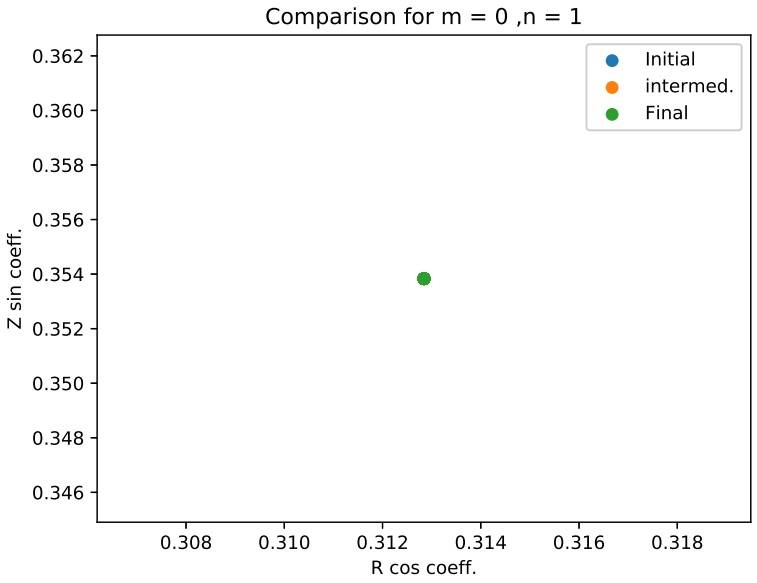




Reducing the initial spread from 5% to 0.1% had seemingly no effect on the distribution of final solutions, which vary significantly.

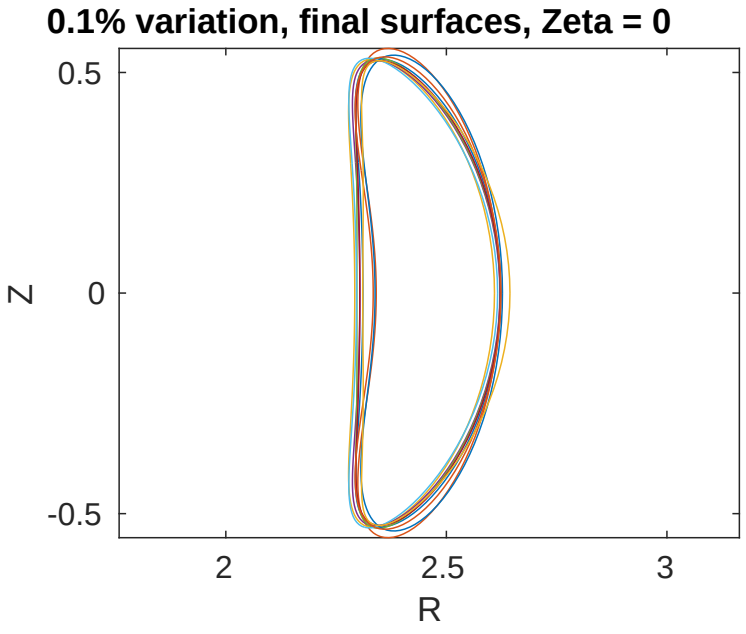
Suggestion 2: Only vary one component

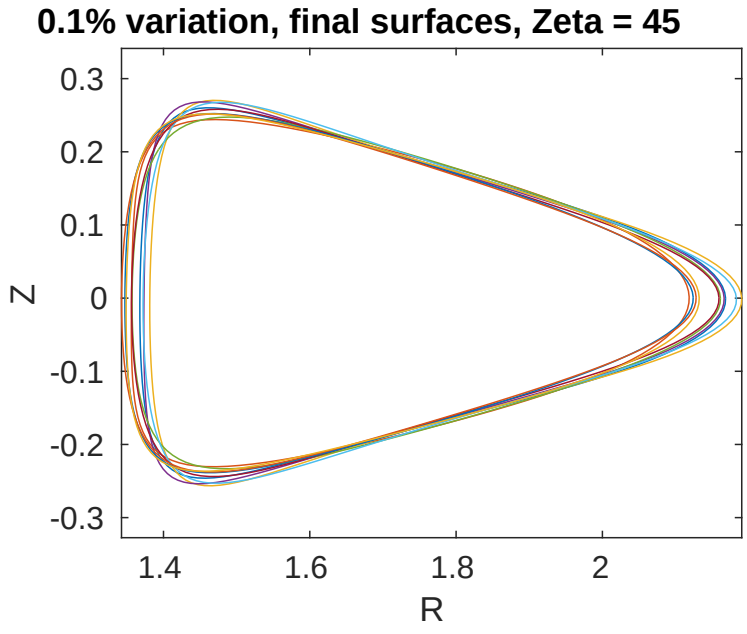




Only the $m = 1$, $n = 0$ terms are initially altered and allowed to be optimized. Solution converges.

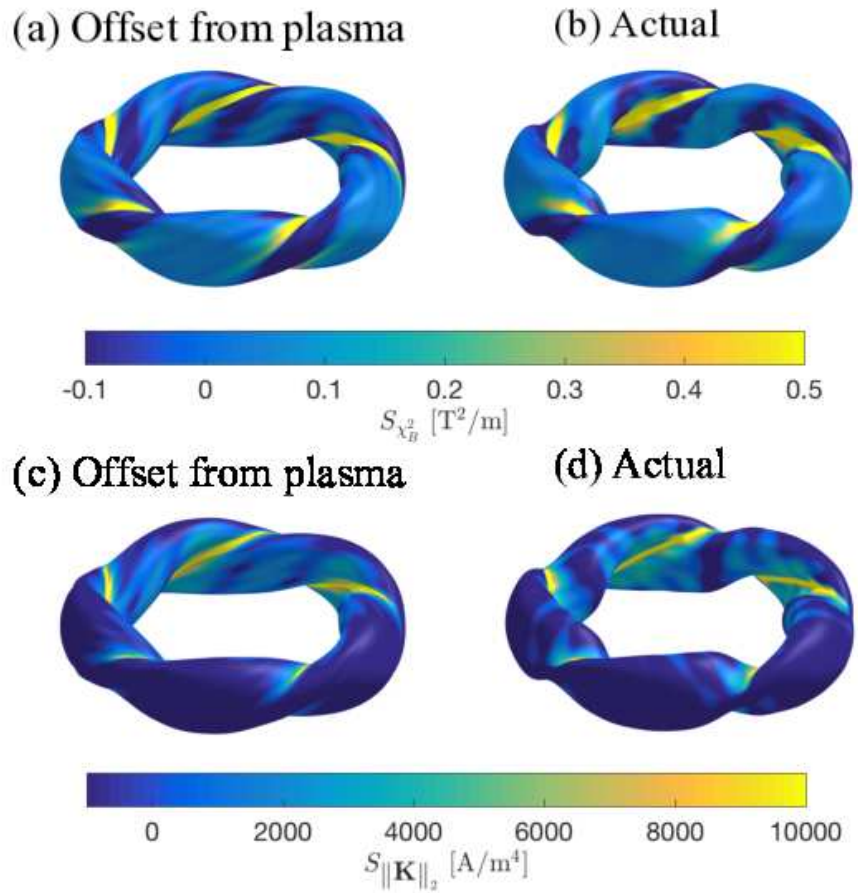
Suggestion 3: Check whether boundaries are actually different in real space





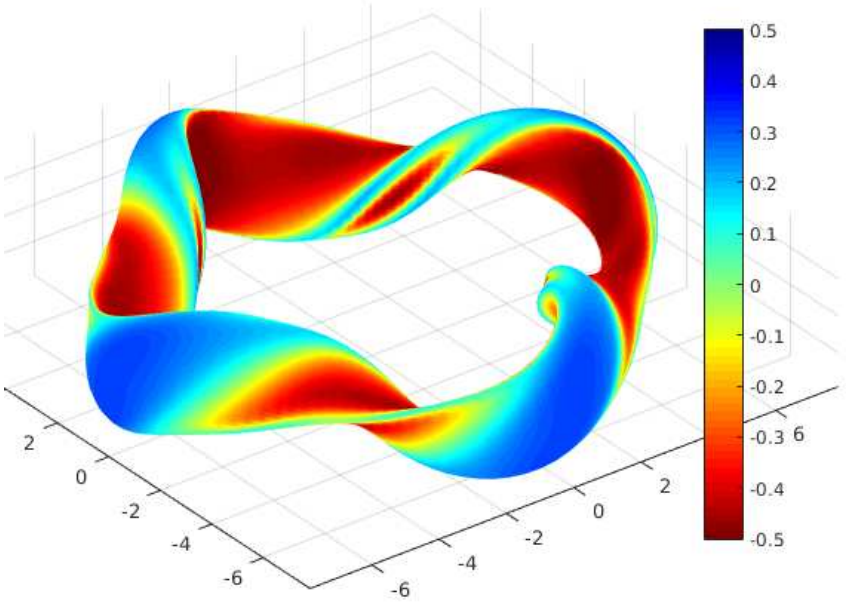
Boundaries are actually different in real space (unfortunately)
Additionally, increasing vmec accuracy, or adjusting LM factor did not yield better results

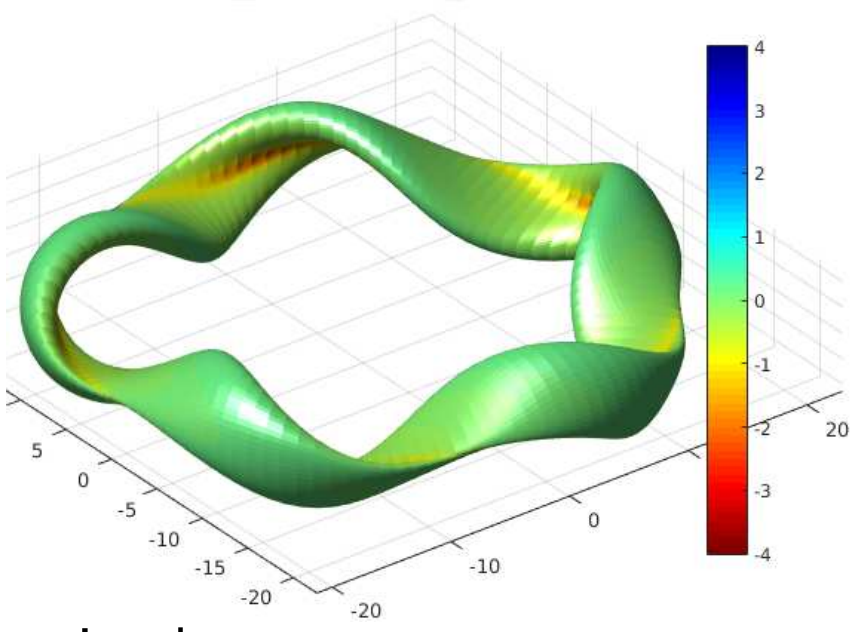
Add metric to directly target concave regions of the boundary



Sensitivity analyses indicate that the concave regions of the plasma are the hardest to produce. They tend to be the limiting factor for minimum coil-plasma distance

HSX and QHS46 have less strong concave regions than other configurations



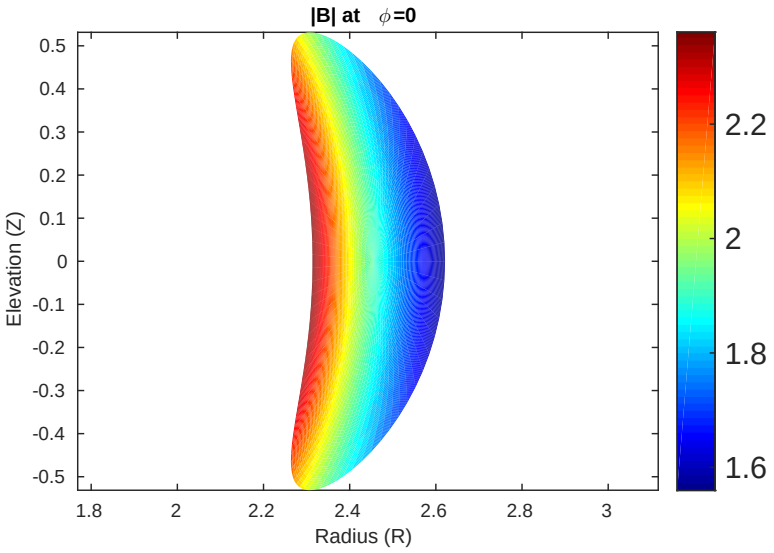


Michael Drevlaks "qhs 8 7" configuration has very strong concavities in the bean cross section, this creates problems for coil generation in that region.

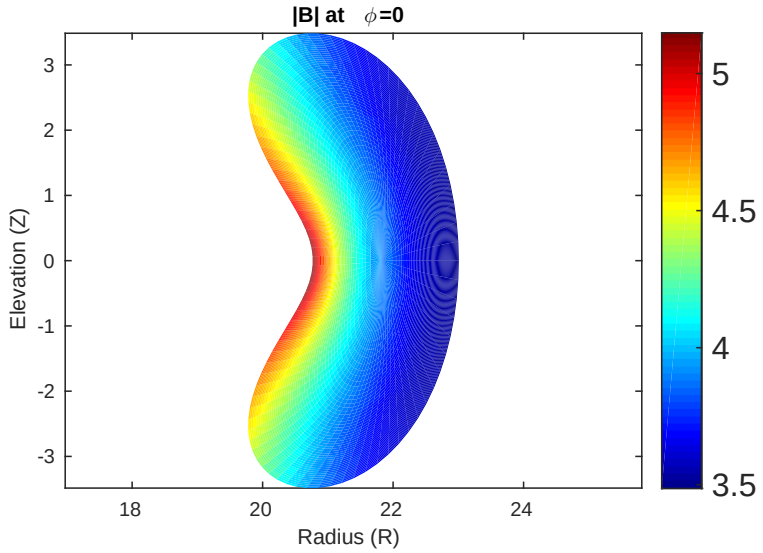
Here the second principal curvature, $P2$ is used as a proxy for concavity.

Bean cross section is the major problem area

QHS46 configuration

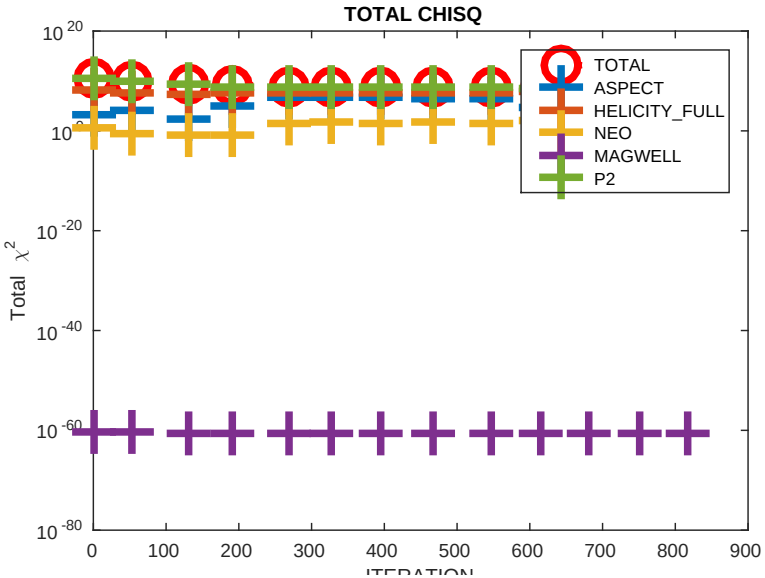


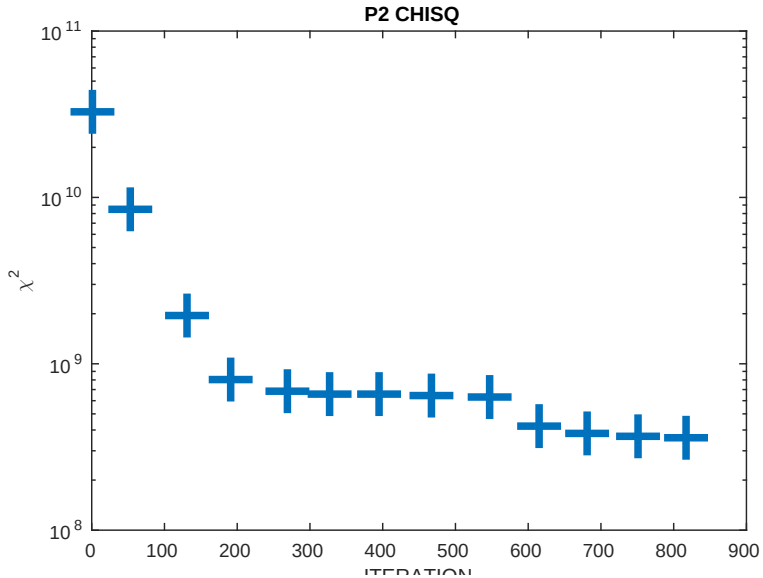
Drevlak 8,7 configuration

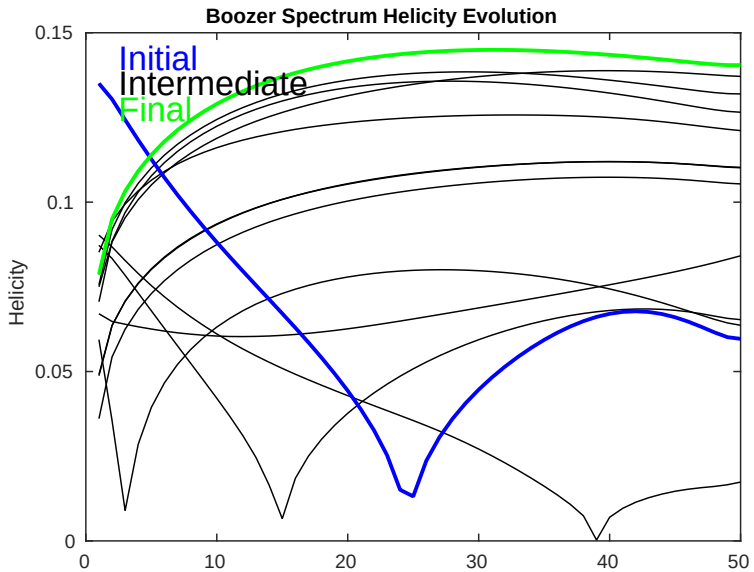


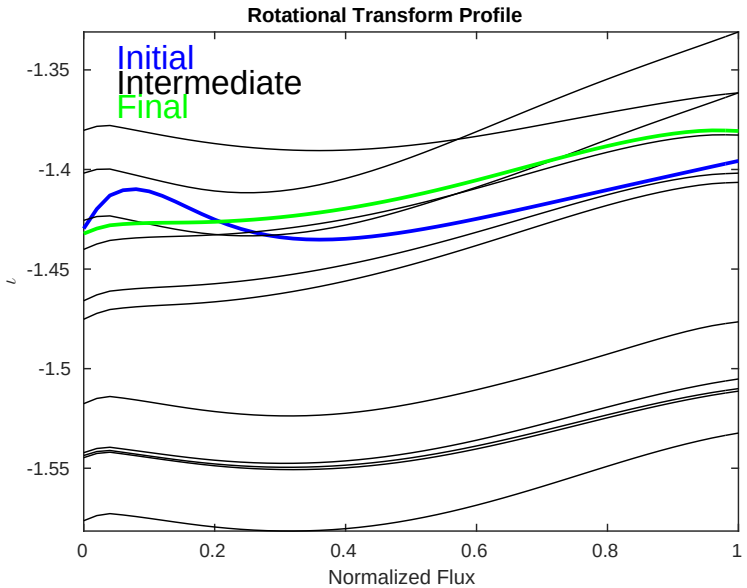
HSX and QHS46 are actually not too bad here. In fact the concavity is so minimal that coil algorithms like FOCUS often have as much trouble in other regions.

Optimizing for *P2* on Drevlak 8,7 configuration

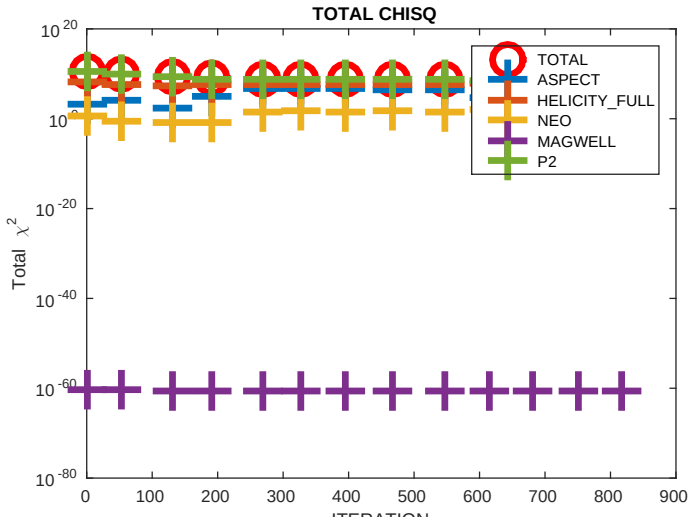


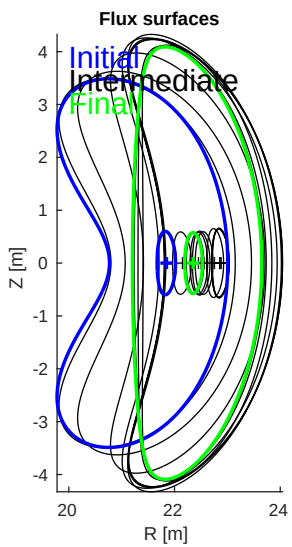




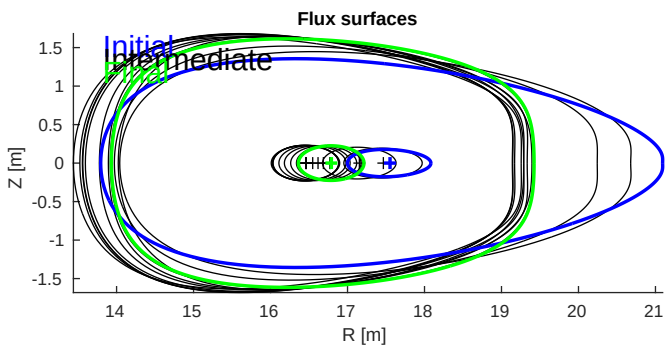


Optimizing for $P2$ on Drevlak 8,7 configuration

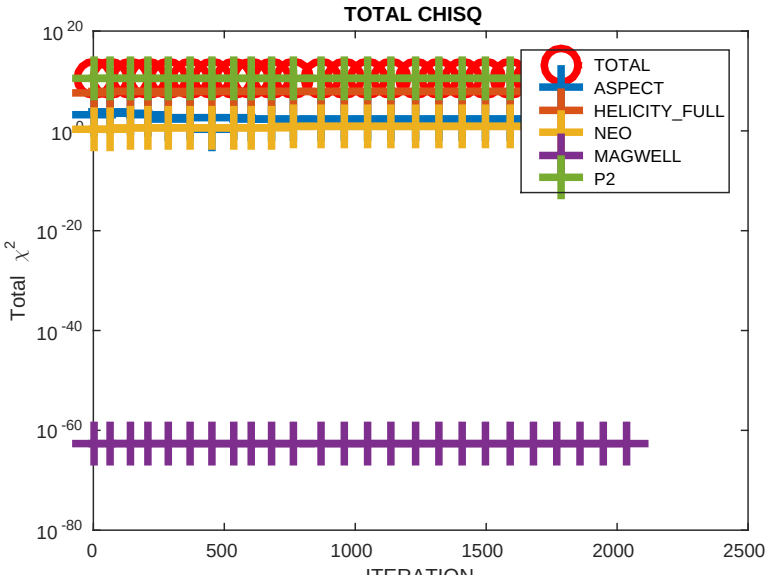


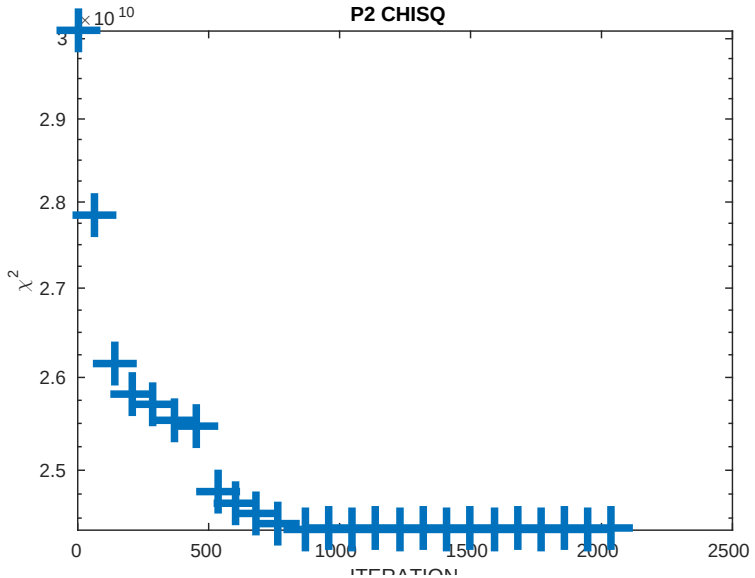


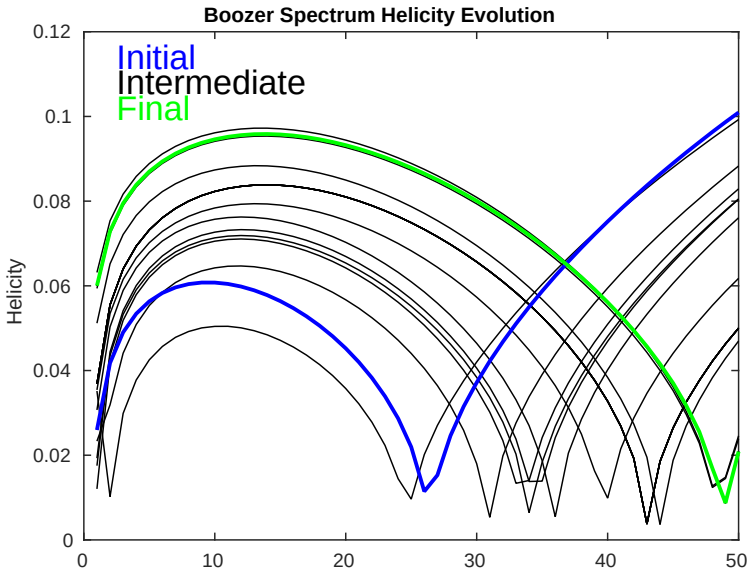
- Significant reduction of the concave region is achieved
- Reduction comes at the expense of Quasi-symmetry

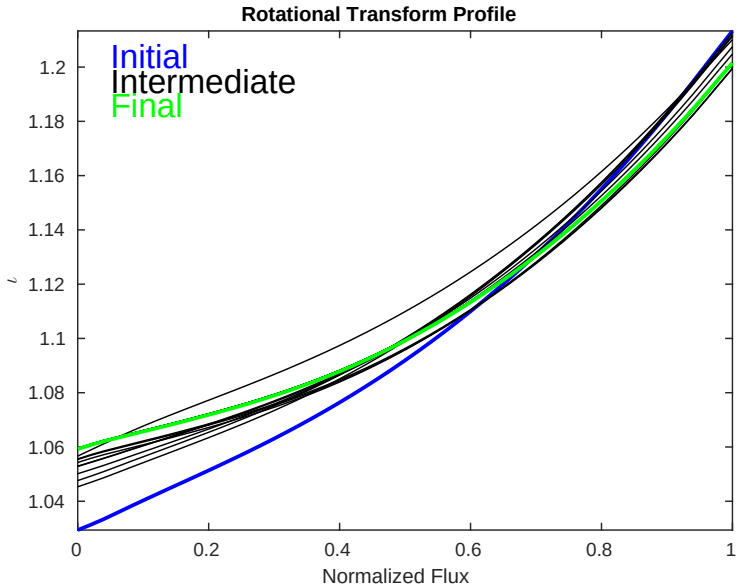


Optimizing for *P2* on QHS46 configuration

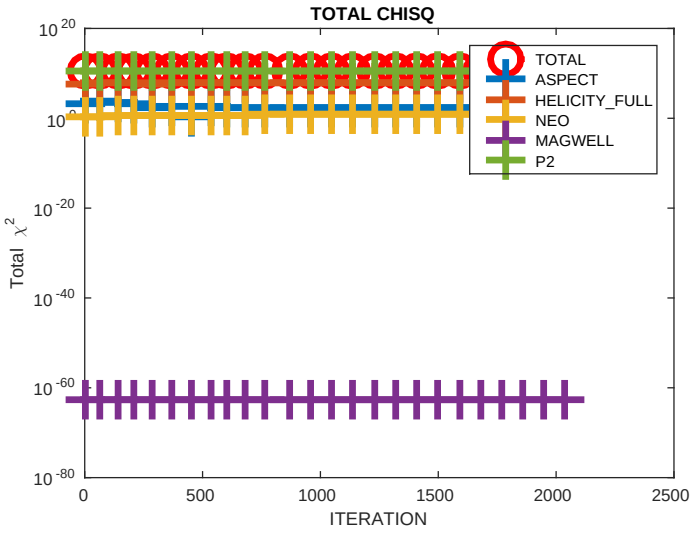








Optimizing for $P2$ on QHS46 configuration



- Modest reduction of concave region is achieved
- It's likely that the QHS 46 configuration and HSX can't really be pushed much further in this aspect