Using STELLOPT/REGCOIL and to find a better winding surface for QHS46
and
Part 2: Discrete coils, free-boundary VMEC MHD solutions and Poincaré plots

With some bug fixes to plots and other typos

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Internal Draft Slides, May 2018

STELLOPT can now manipulate the set of Fourier coefficients that describe the coil winding surface

- Winding surface is specified by a set of Fourier coefficients in toroidal space $(\theta,\ \zeta)$
  - Coordinate system
    - Superimpose Cartesian (X, Y, Z) and spherical (R, ζ, Z)
    - $\theta = \text{atan}\left(\frac{Z}{R}\right)$ Poloidal angle (in the $\mathrm{R-Z}$ plane)
    - $\zeta = \text{atan}\left(\frac{Y}{Z}\right)$ Toroidal angle (in the $\mathrm{X-Y}$ plane)
  - The surface (2D manifold) is defined by the locus of point given by

$$R(\theta,\zeta) = \sum R_{m,n,c}\cos(m\theta + n\zeta) + \sum R_{m,n,s}\sin(m\theta + n\zeta)$$

$$\mathrm{Z}(\theta,\zeta) = \sum Z_{m,n,c}\cos(m\theta + n\zeta) + \sum Z_{m,n,s}\sin(m\theta + n\zeta)$$

- Stellarator symmetry is enforced for now
  - With stellarator symmetry, many modes must be 0: All: $R_{m,n,s}$ All: $Z_{m,n,c}$, and some (m,n) combinations
  - The ability to allow non-stellarator symmetry has been implemented, but not tested

- STELLOPT varies the coefficients and improves the quality of B_{norm} on the target surface

  - Initial guess: A winding surface with a uniform separation (See previous presentations)
  - Optimal winding surface: For QHS46, the initial trend is to make the winding surface:
    - Vertically elongated
    - Closer to the plasma, radially

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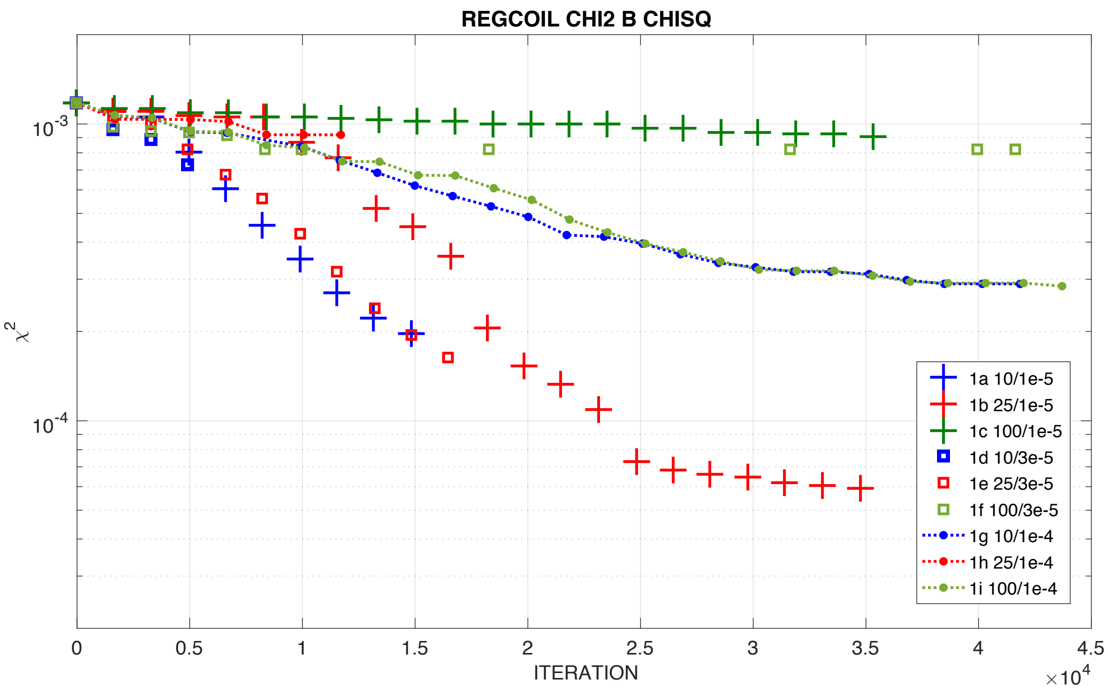
STELLOPT finds a better solutions (smaller residual error, reduced B_{norm} on the target plasma surface) when manipulating the Fourier coefficients. An improvement by one order of magnitude compared to the uniform winding surface.

- Initial guess was a winding surface with uniform separation, K_{RMS}= 2.82e6 A/m
- 1618 total harmonics were allowed to be varied
- 1618 identical targets of $\chi^2_{B_{norm}}$
- Behavior exhibits a dependence on L-M optimizer run parameters of FACTOR / EPSFCN
- Time limit reached
  - Cases b, c, f, g, i
- "algorithm estimates that the relative error between x and the solution is at most xtol"
  - Cases a, d, e, h



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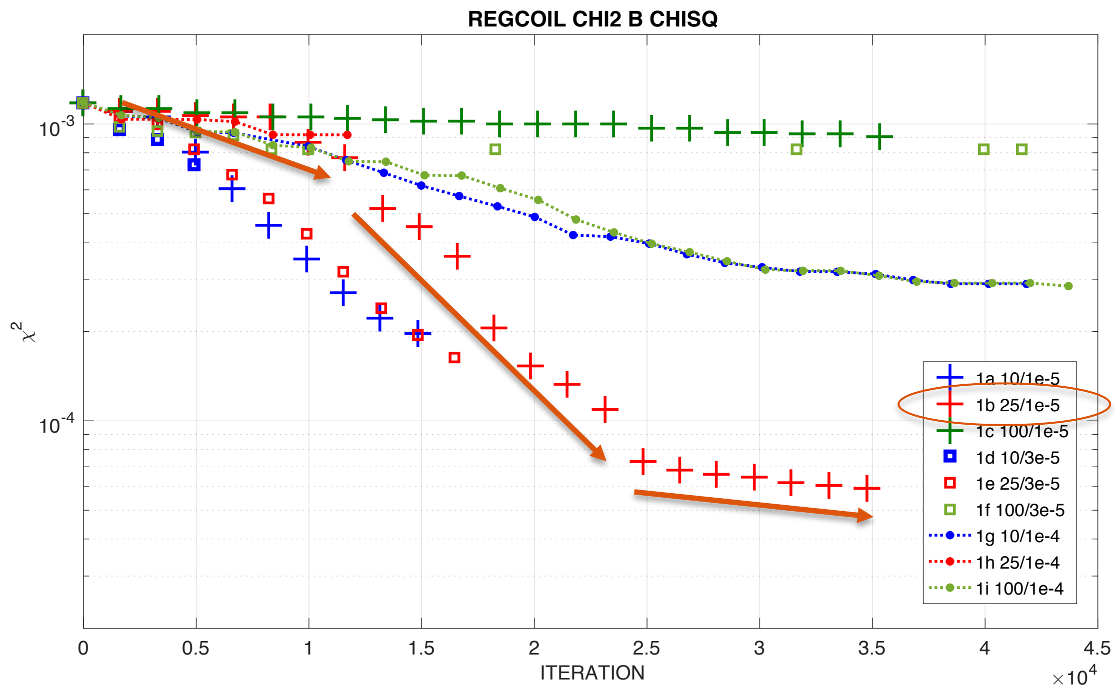
The next 21 slides show the evolution of the new ‘optimal’ solution during case 1b with FACTOR/EPSFCN = 25 / 1e-5

- This is the case shown in red ‘+’ symbols, see arrows
- The first slide will explain what is being shown in each subplot
- There are 21 set of plots, one for each new solution



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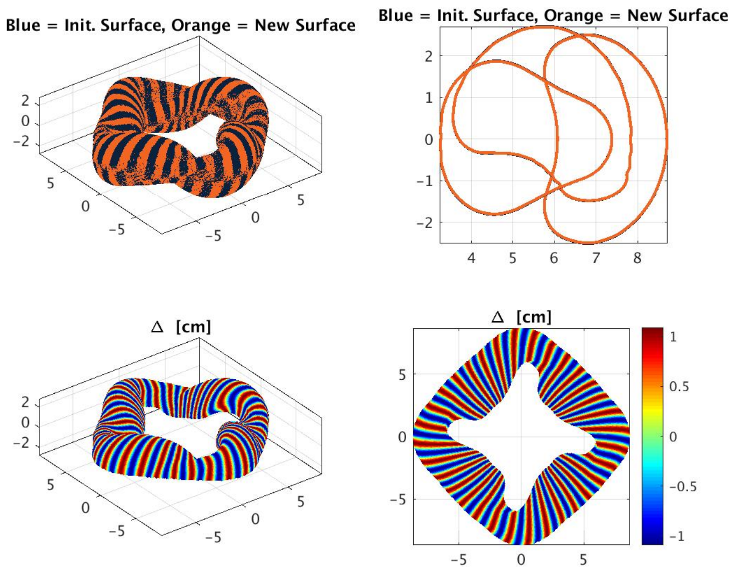
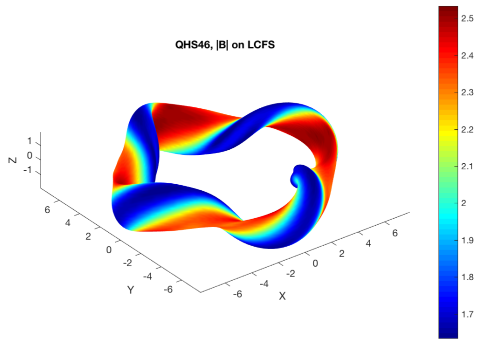
As an example, the first step (but don’t dwell on it)

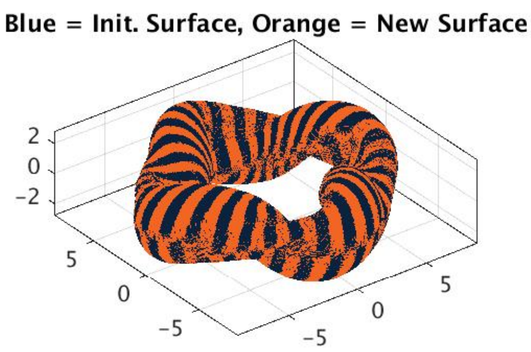
- X, Y, Z axis units are meters
- Upper Left: The original winding surface (in black) and the new winding surface (orange) are superimposed on the same axes. This allows a simple ‘binary’ eye inspection to see the gross features of the surfaces
- Upper Right: Planar slices of the winding surface at $\zeta = (0°, 45°, 90°) * \frac{360°}{NFP}$
- Lower left and right: The distance (in cm) from the new surface to the original surface, multiplied by the sign of the change in the vertical (Z) –direction

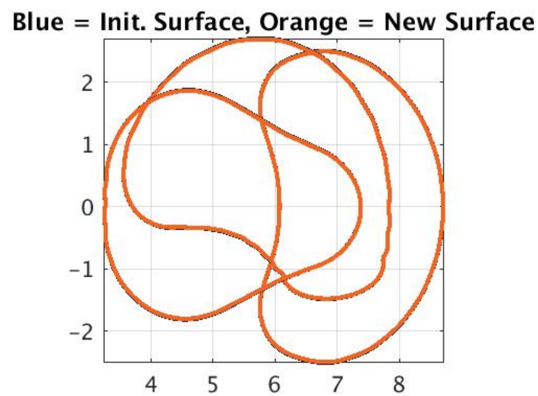


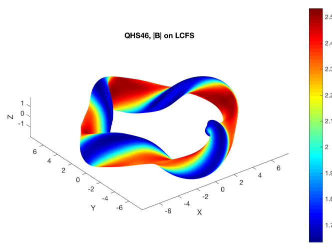
- Original target |B| on target LCFS also shown for reference
  - This is not a ‘self-consistent’ |B|

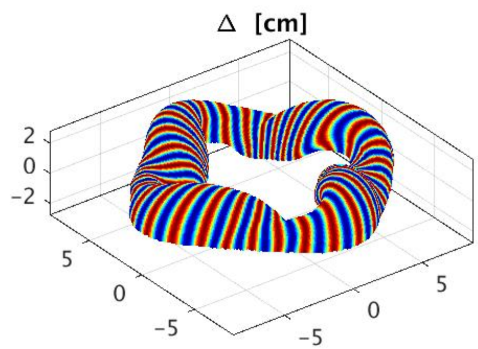


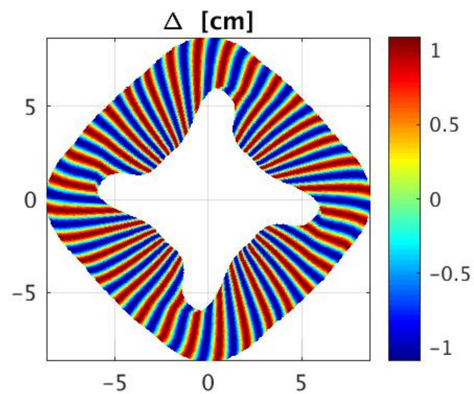
Iterative solution #2, The first step











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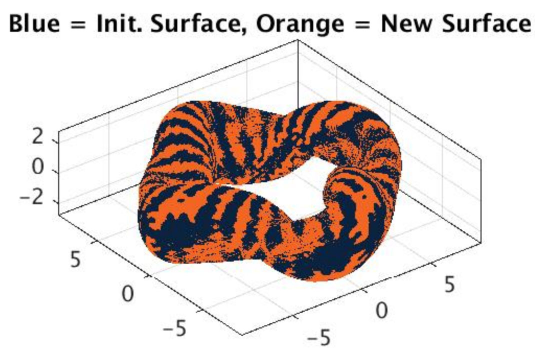
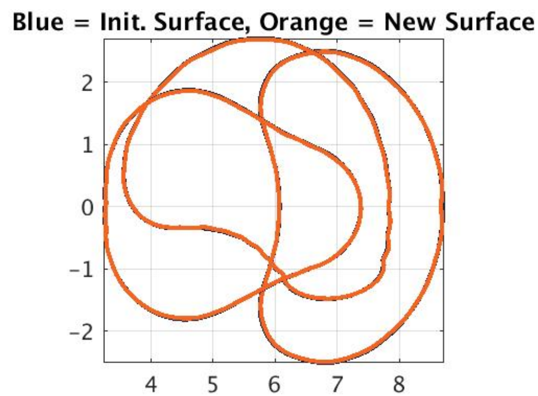
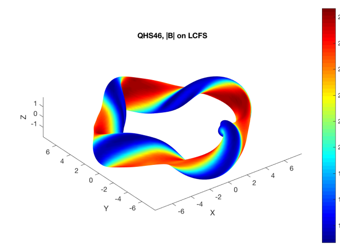
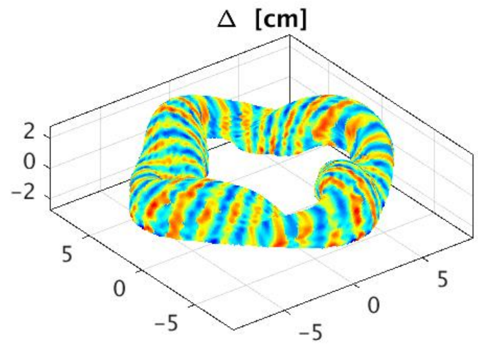
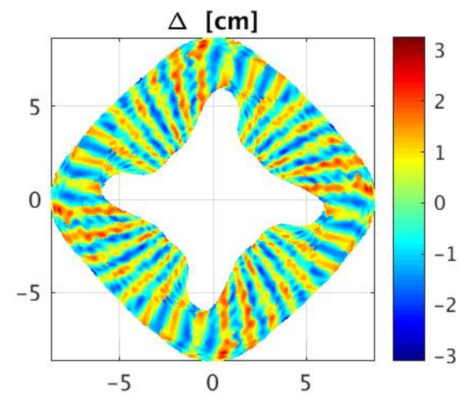
Iterative solution #3



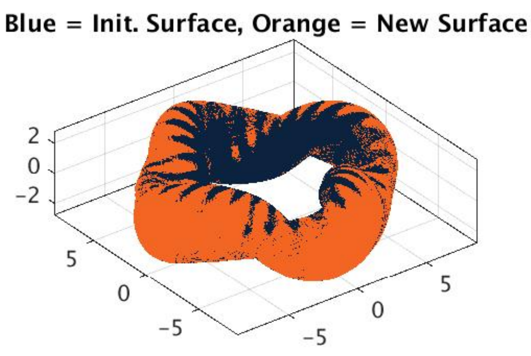


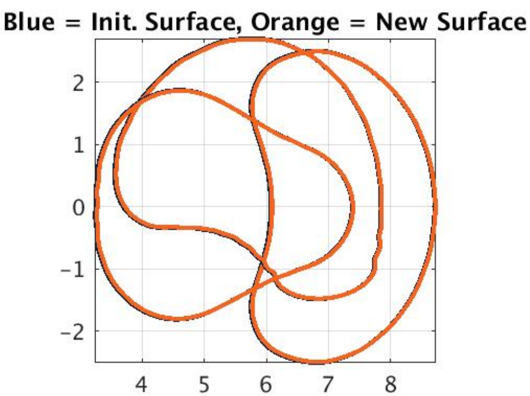


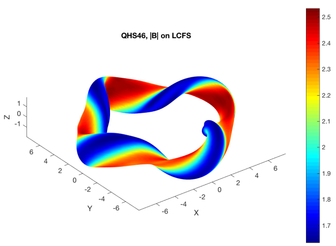


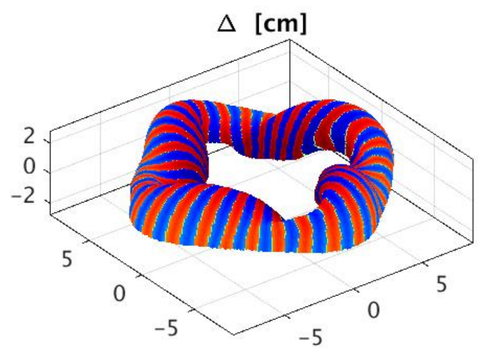


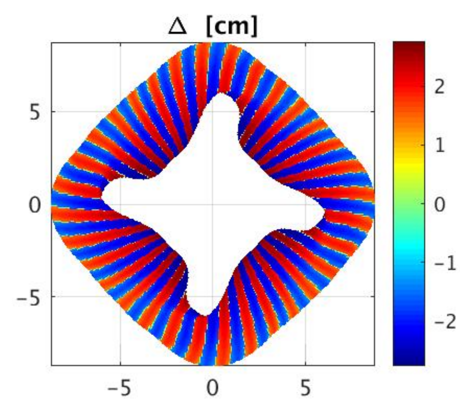
Iterative solution #4



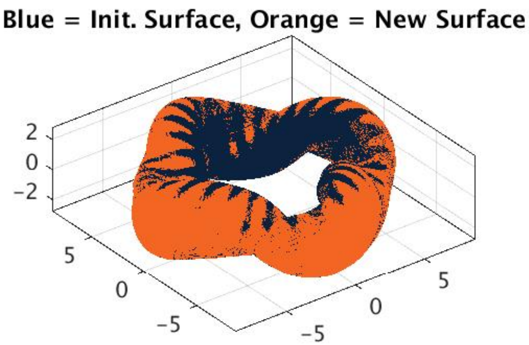


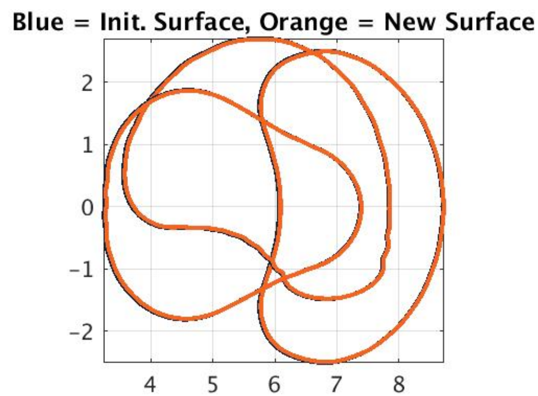


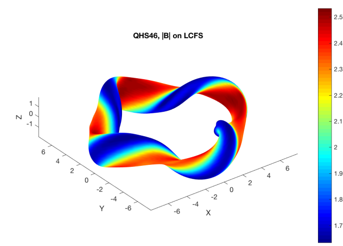


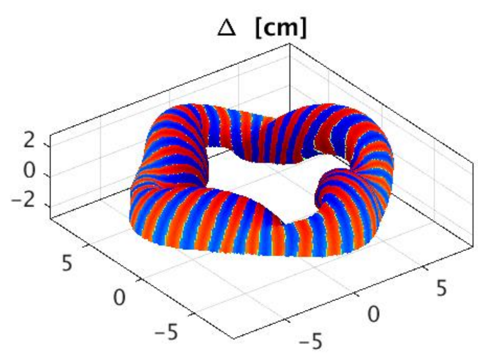


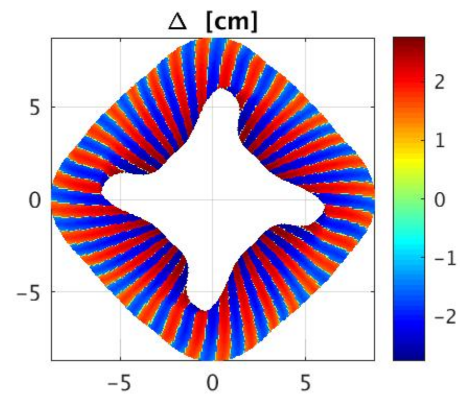
Iterative solution #5



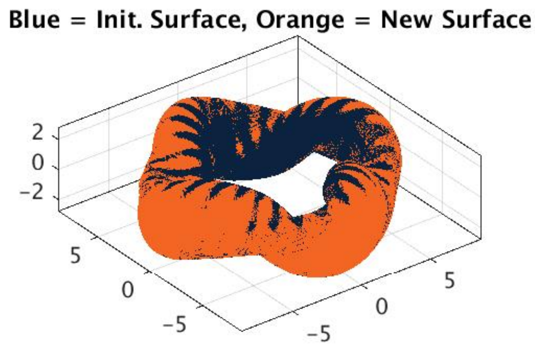


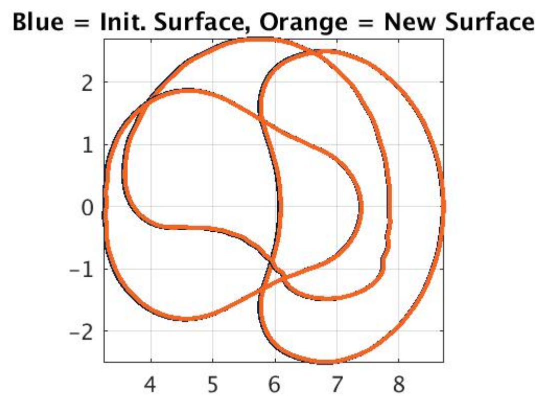


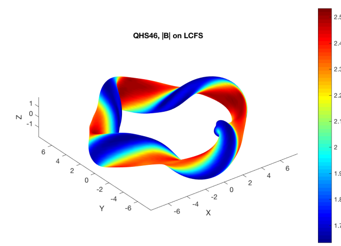


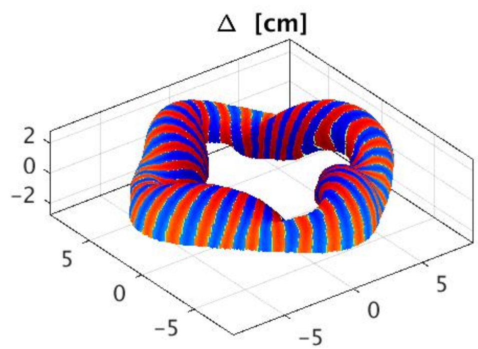


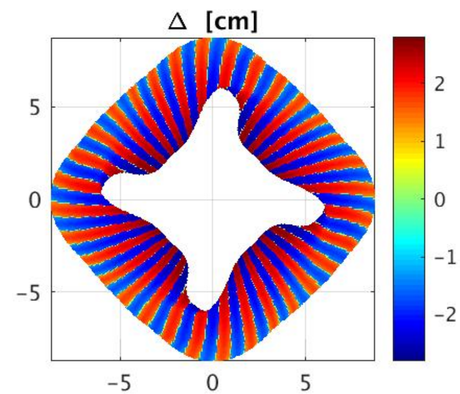
Iterative solution #6



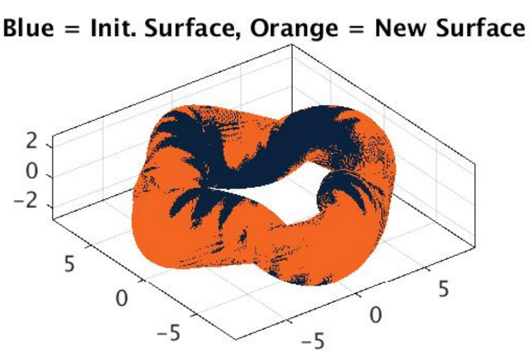


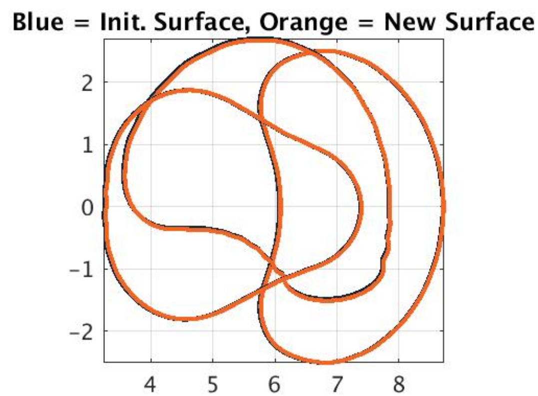


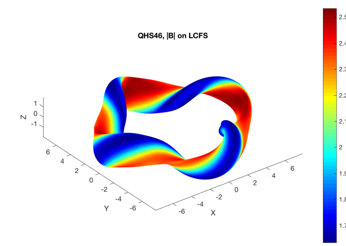


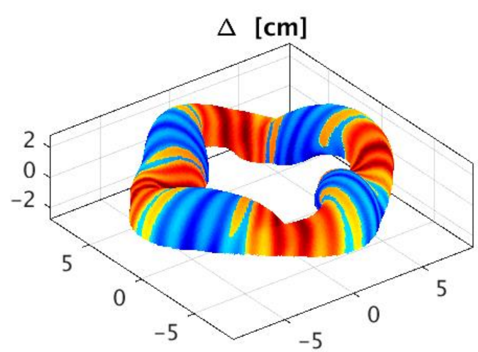


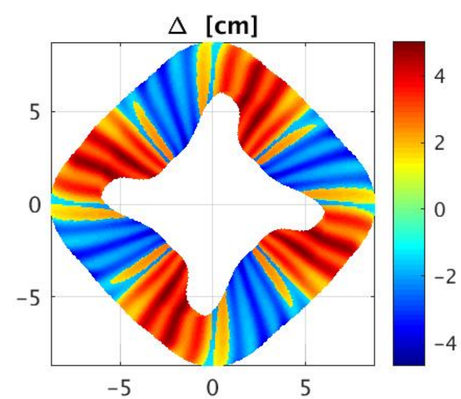
Iterative solution #7











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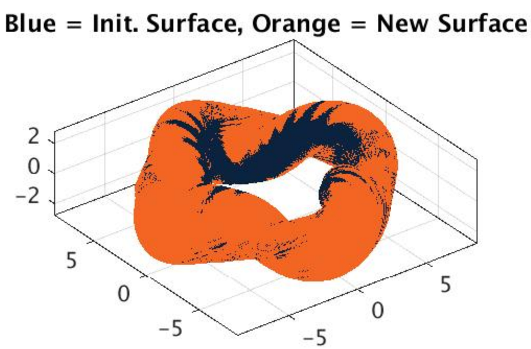
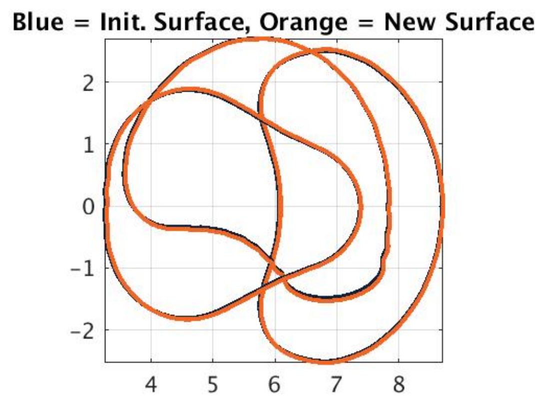
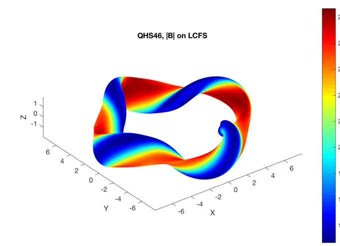
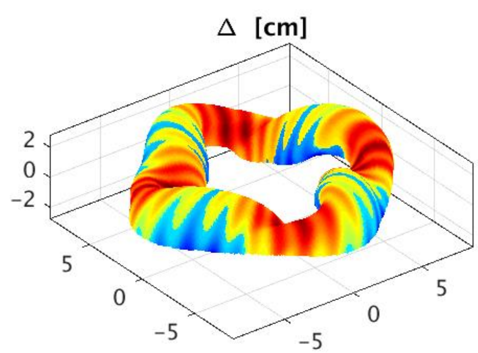
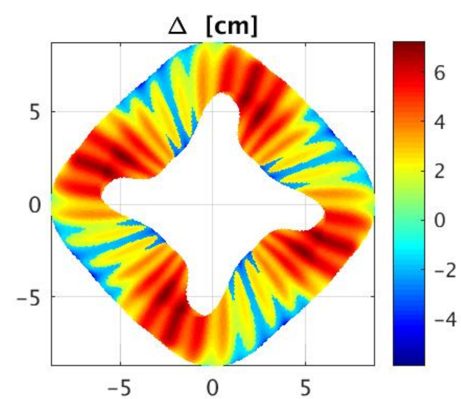
Iterative solution #8



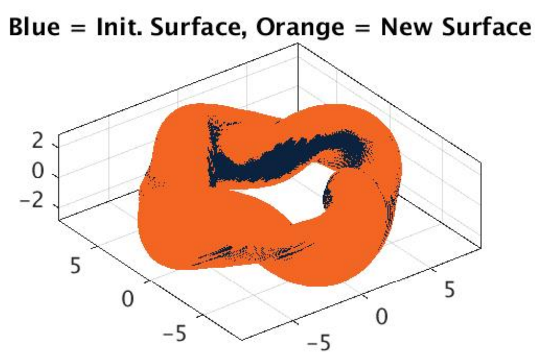


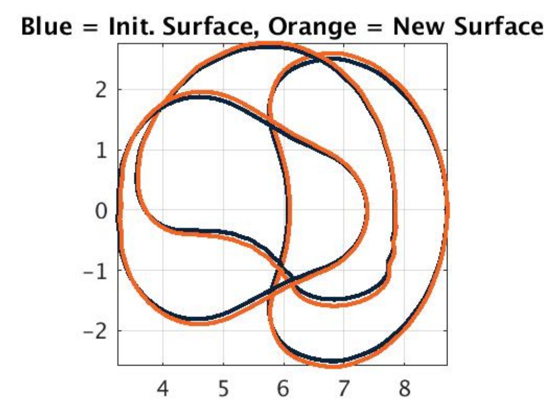


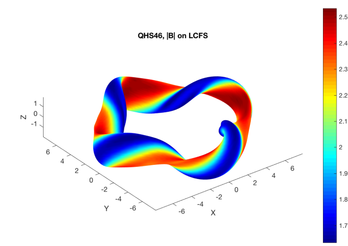


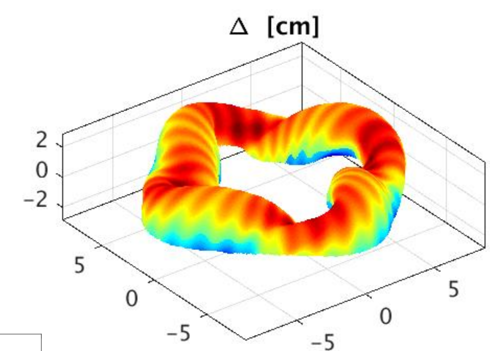


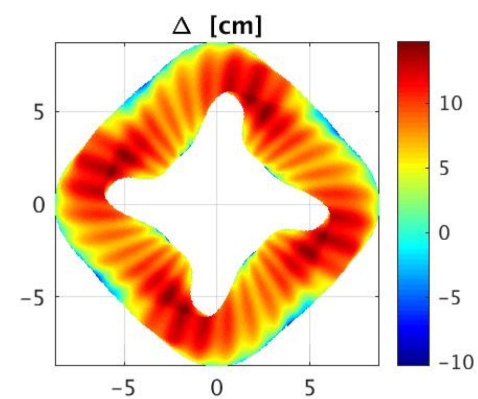
Iterative solution #9: A new rate of descent begins now; Surface will become narrower and taller.

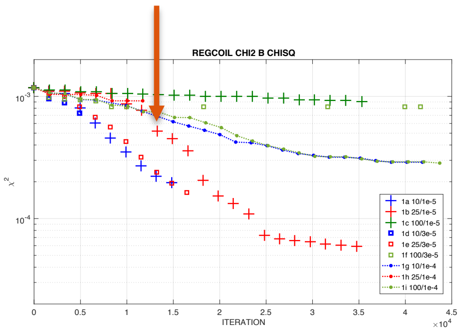




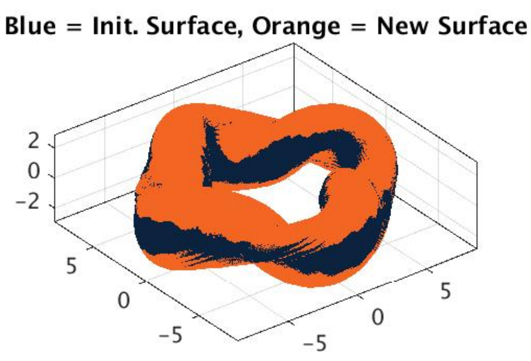


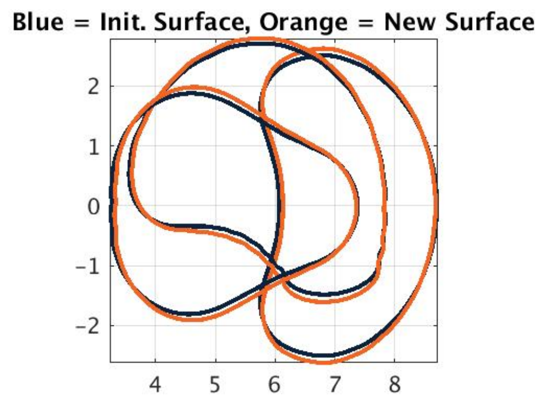


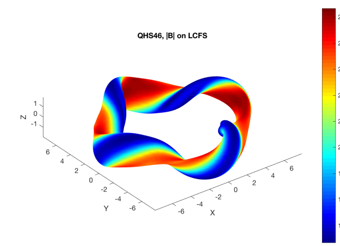


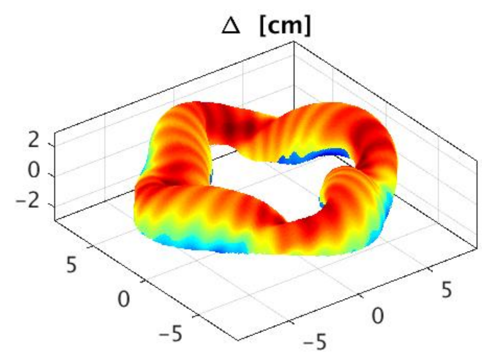


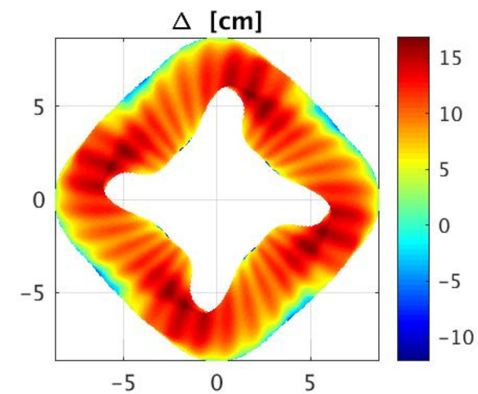
Iterative solution #10



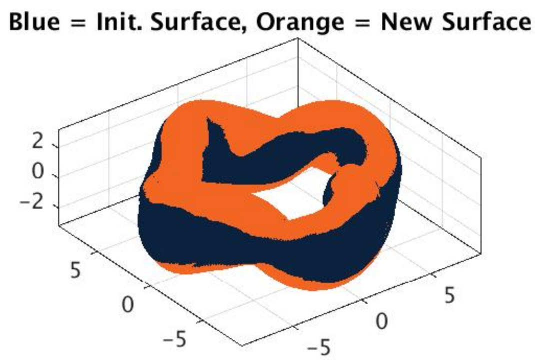


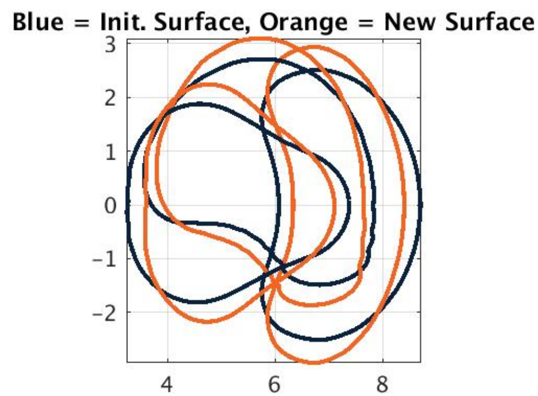


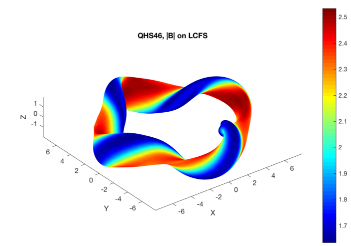


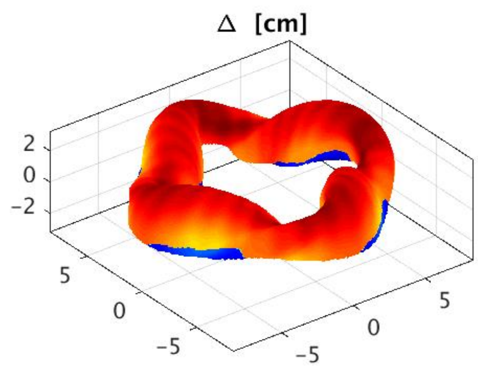


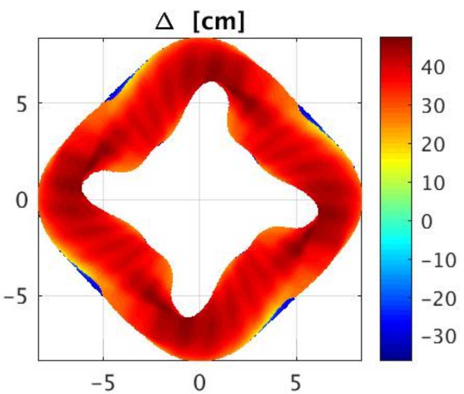
Iterative solution #21











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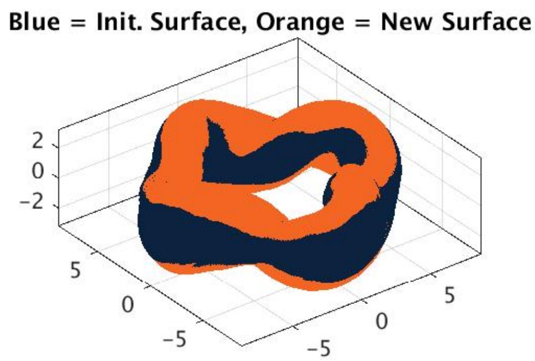
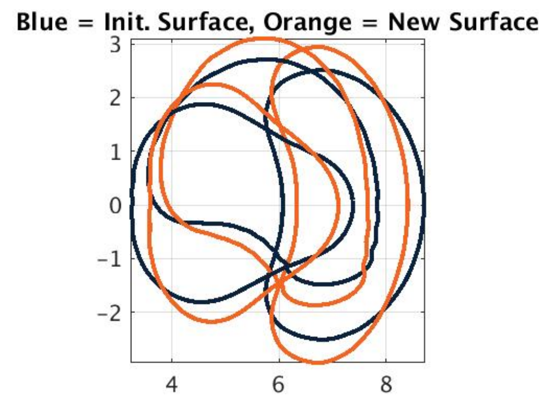
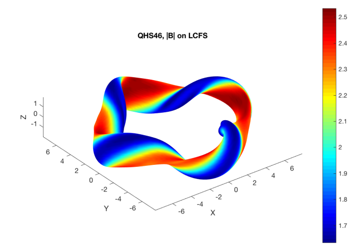
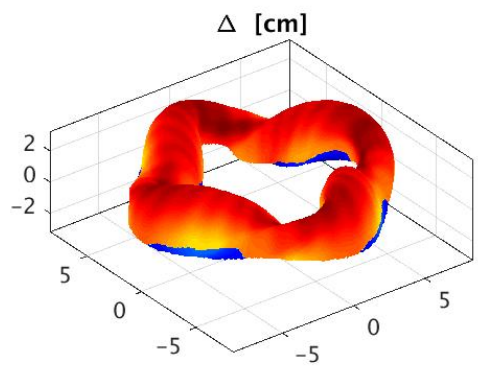
Iterative solution #22: (24x32 cpu-hour time limit)

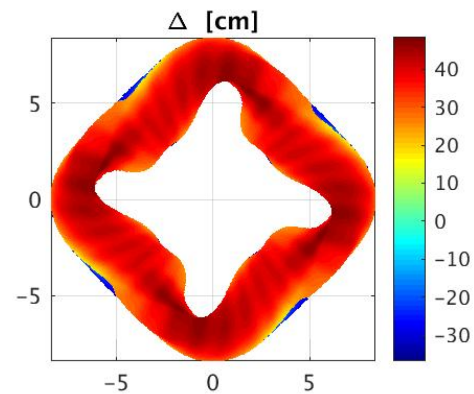
- The solution after the last iteration is teller and narrower than the starting position
- Recall: The starting position was a ‘uniform distance’ away from the plasma boundary











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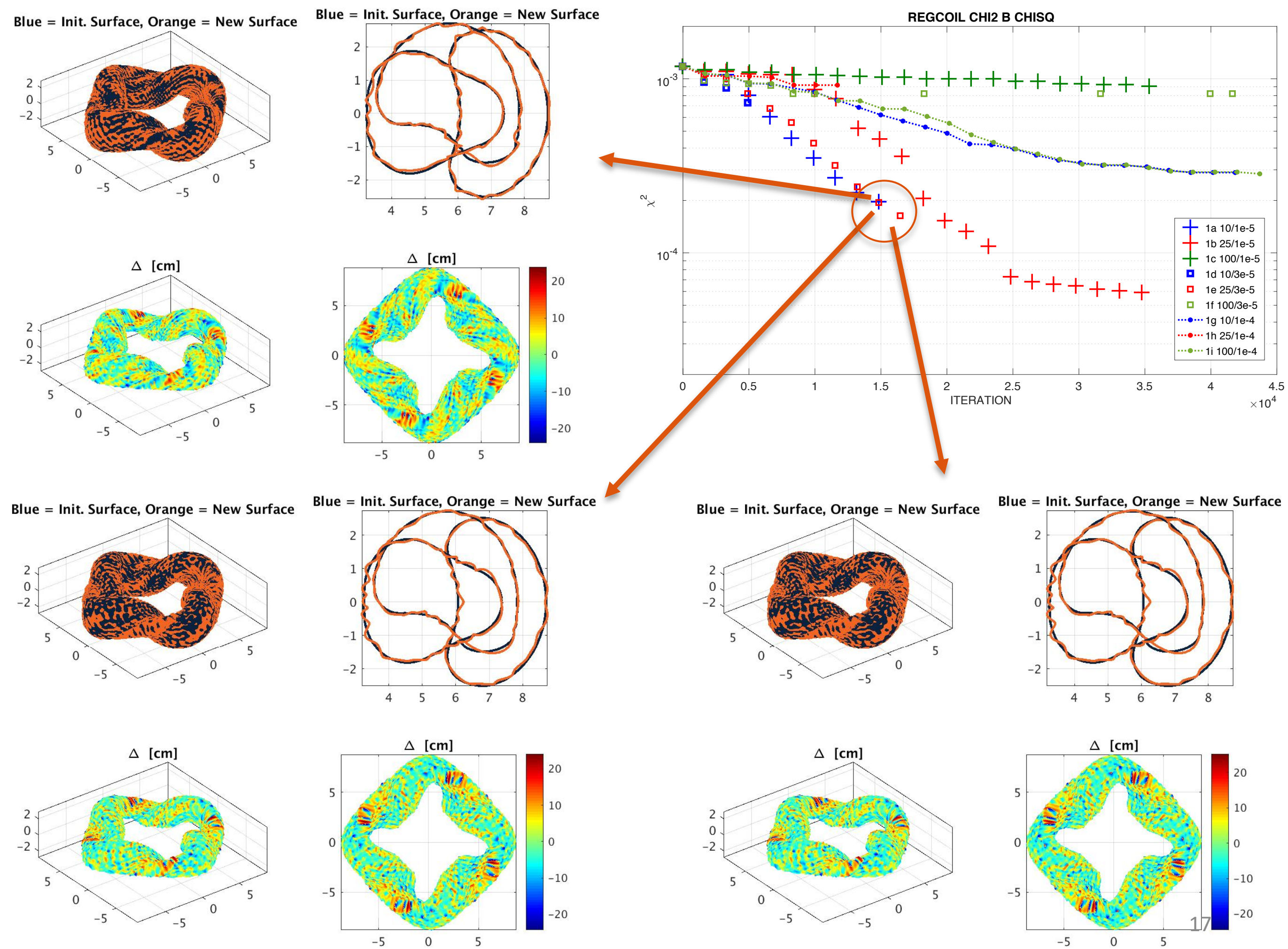
The optimizer runs that demonstrated the most rapid descent A) look a little different, B) are still evolving, and C) terminated because the solver was in a local minimum (I think)

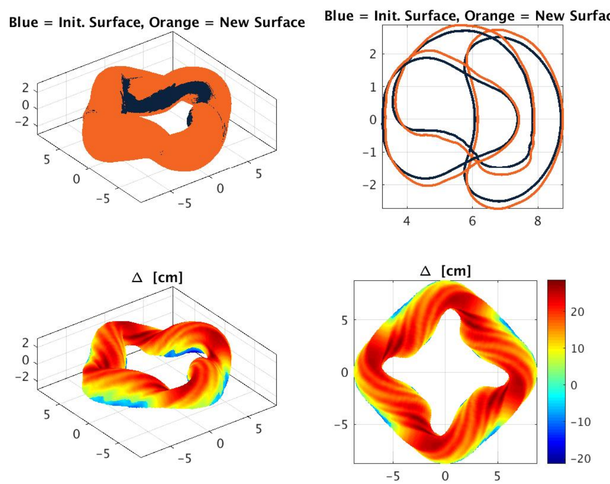
- The surfaces of the two branches that descend more rapidly, but are truncated, look a little different
- blue +'s (upper left)
- red squares (lower left)

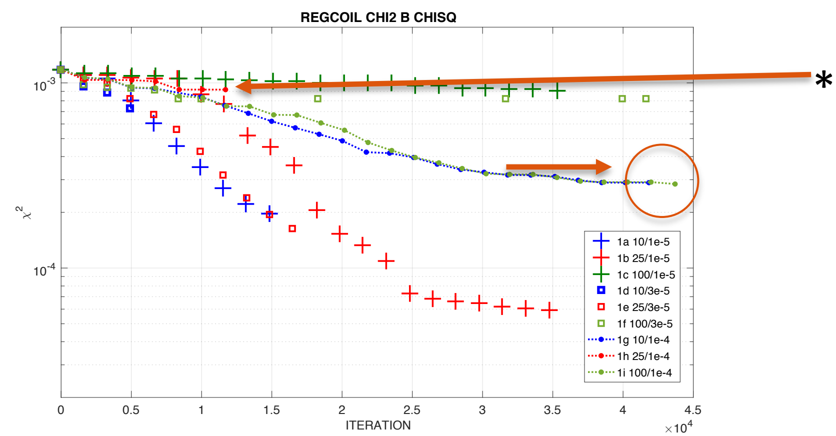
- The red square descent path has one extra step – see lower right plots

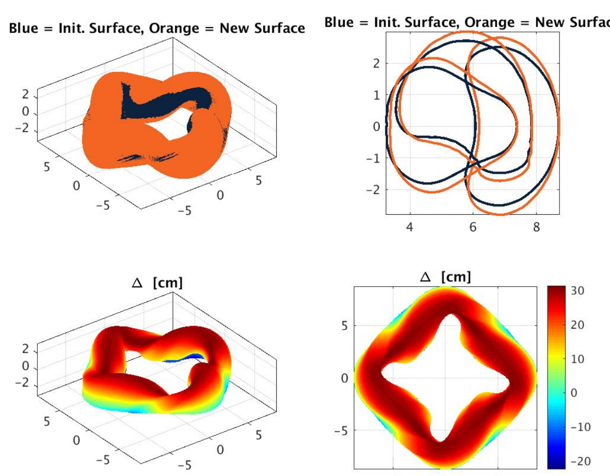
➢ **The solver got stuck in a local minimum?**
  ➢ "algorithm estimates that the relative error between x and the solution is at most xtol"

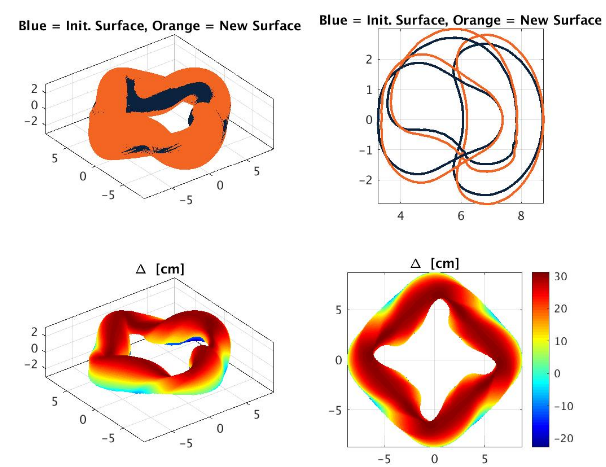


The optimizer branch that descends more slowly looks vaguely like the ‘best’ case: Taller, **but less narrow**!









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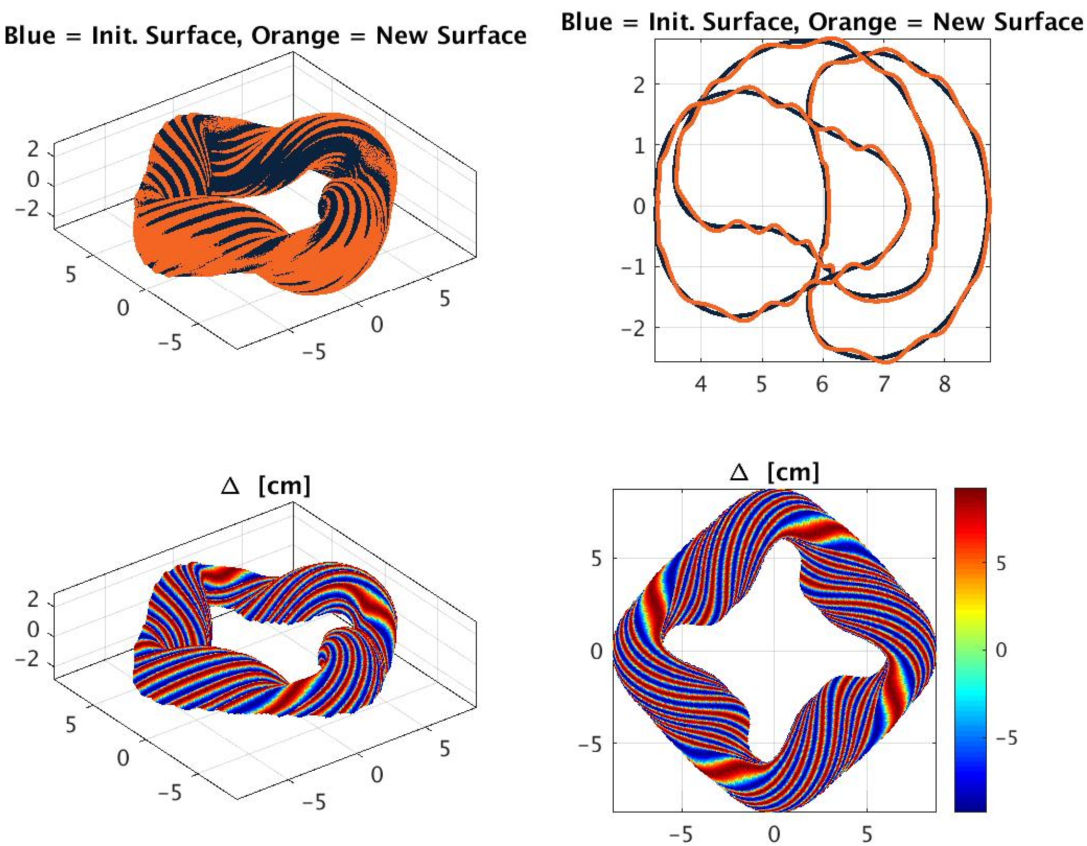
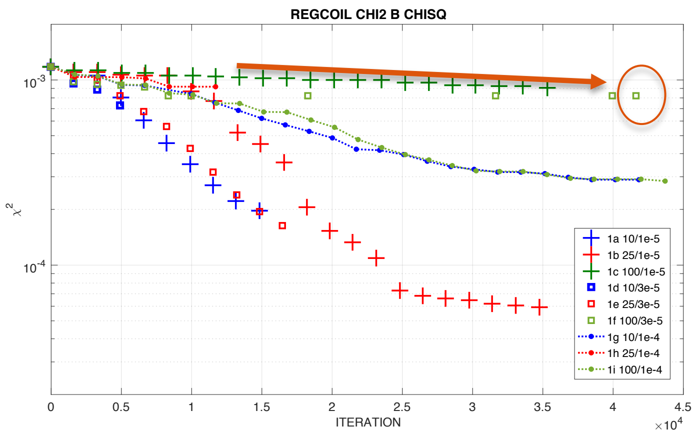
Helical Stripes: The runs that descend very slowly has very corrugated surfaces.





Discrete coils, VMEC 'surfaces' and Poincaré plots

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Coils, MHD equilibrium and Boozer spectrum for two different winding surfaces: ‘Uniform distance’ and ‘Fourier/Optimized’

- Cutting coils, calculating the vacuum VMEC MHD solution and Boozer spectrum
- Generate coilsets for two different winding surfaces
  - The 1st winding surface is ‘uniformly displaced’ 85.3 cm from the plasma
    - Shown in a previous Group Meeting: March 23, 2018 @ https://wistell.engr.wisc.edu/?page_id=156
  - The 2nd winding surface is the one found by STELLOPT with Fourier Series variations
    - shown in detail on slides 4-26 on this presentation
    - Difference approx. +(40-45)/-(30-35) cm in displacement of winding surface
  - Coilsets explored consist of a discrete number of unique coils
    - Uniform winding surface: 4, 5, 6 (Surfaces are not great)
    - Fourier/Optimized winding surface: 3, 4, 5, 6 (VMEC converged with even only 3 coils/FP)
- VMEC(+post) are run for each of the cases
  - Free-boundary VMEC (using the coils via an ‘mgrid’)
  - Boozer spectrum (xbooz_xform)

➢ **FOR NOW, I only show the case with 6 coils**

➢ **Other cases are interesting, but the ‘best Boozer spectrum’ results to date have been found with 6 coils per field period**

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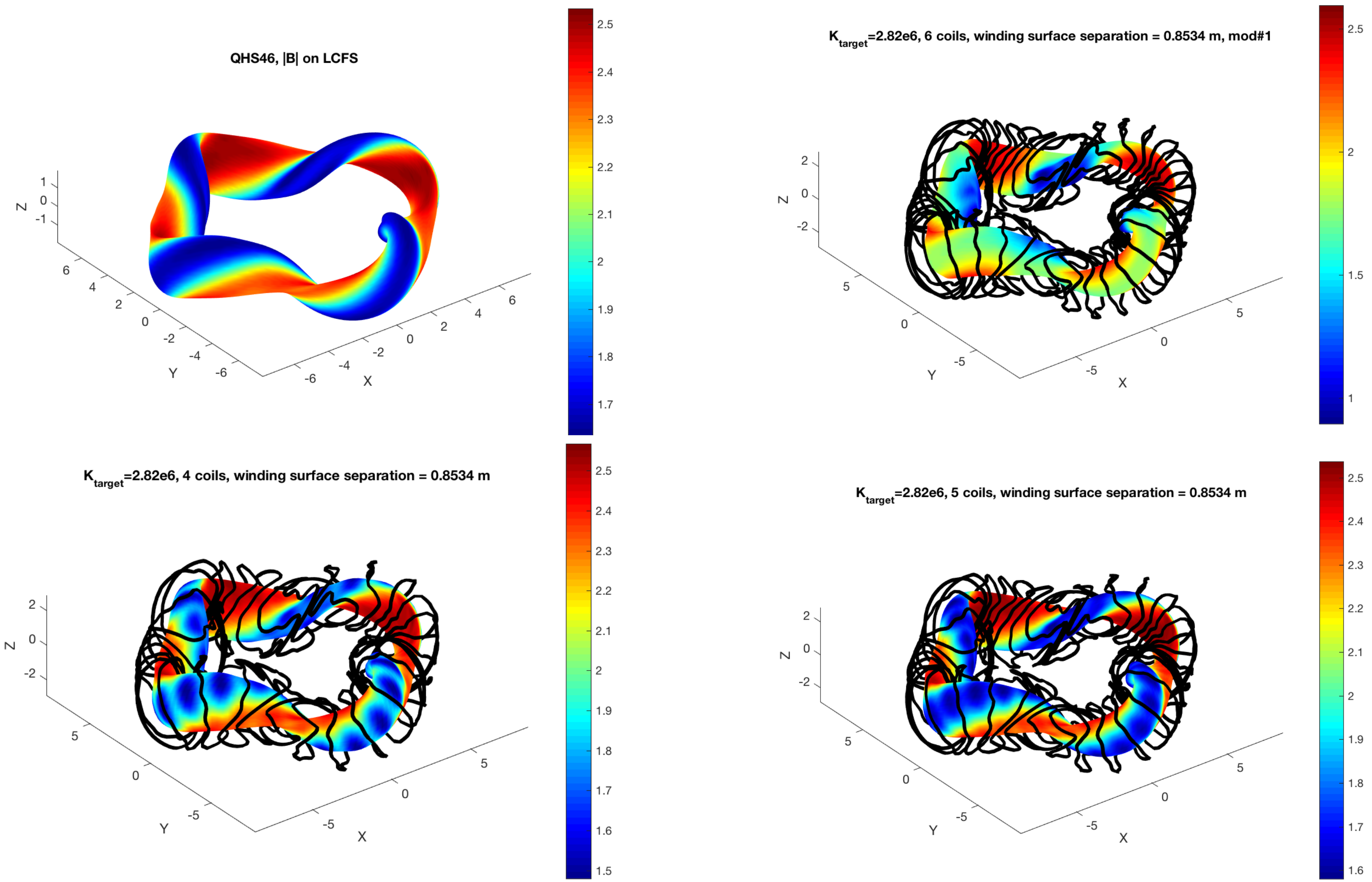
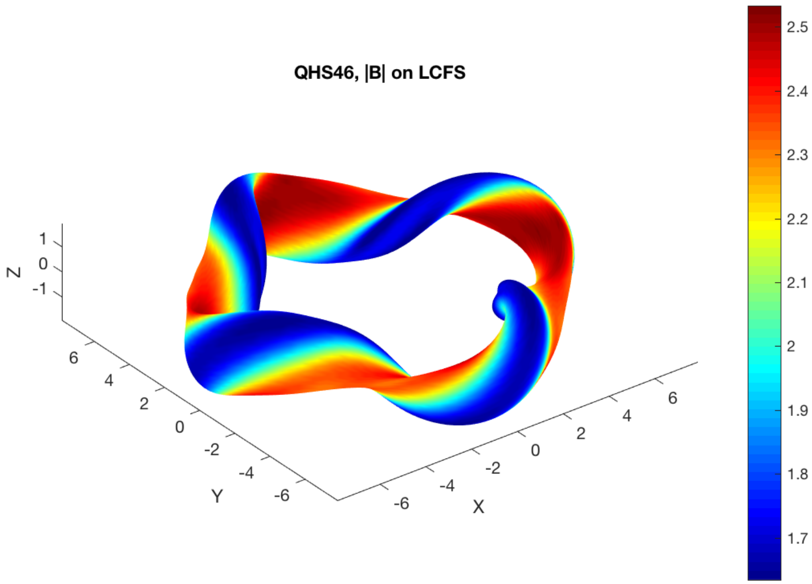
Slide labels

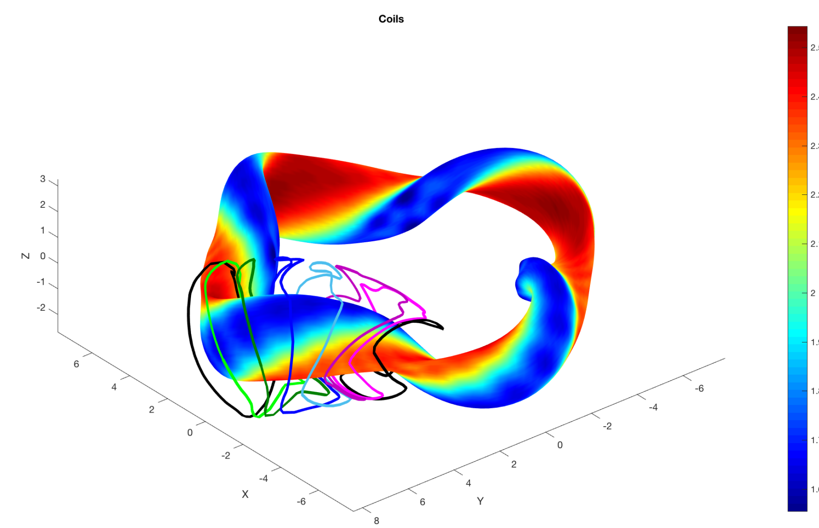
- Slide 23
  - Mod B on LCFS for: QHS46 and Coil set with {4, 5, or 6} coils on uniformly displaced surface
  - Lots of tight twists, closely-fitted high-curvature sections. Not great results
- Slide 24
  - Mod B on LCFS for: QHS46 and with 6 Coils / field period on the Fourier/Optimized surface
  - Also see the interactive Matlab figure
- Slide 25: Different views of the coils
- Slide 26: Poincaré plot with field line following and VMEC surfaces for vacuum provided by 6-coil / field period model

4, 5, or 6 Coils / field period
Uniform winding surface, 2.82e6 A/m



6 Coils / field period
Fourier/Optimized winding surface
2.82e6 A/m on winding surface; 1.5 MA in each coil





Also see the interactive matlab figure:

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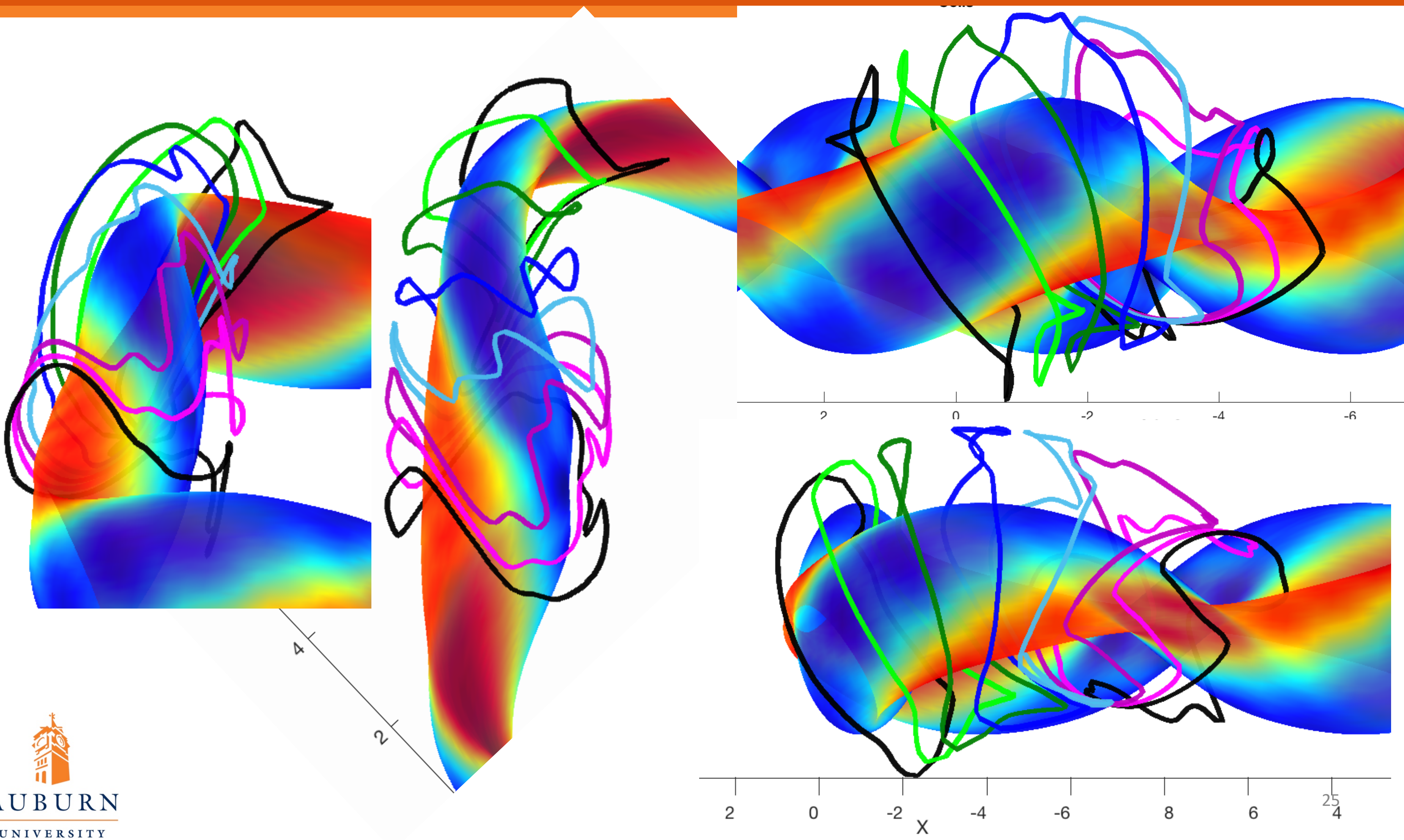
6 Coils / field period
Fourier/Optimized winding surface



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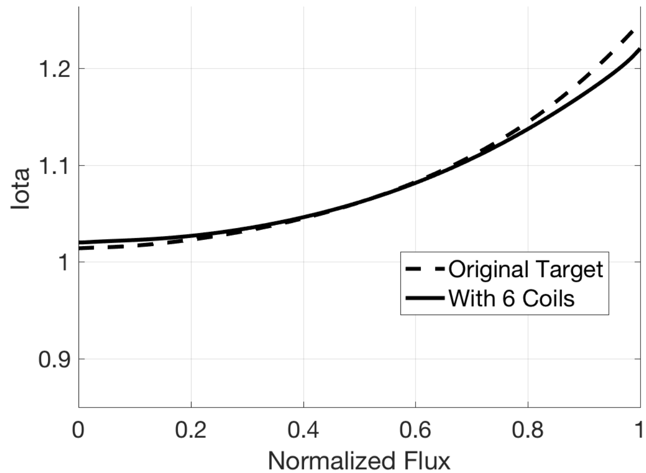
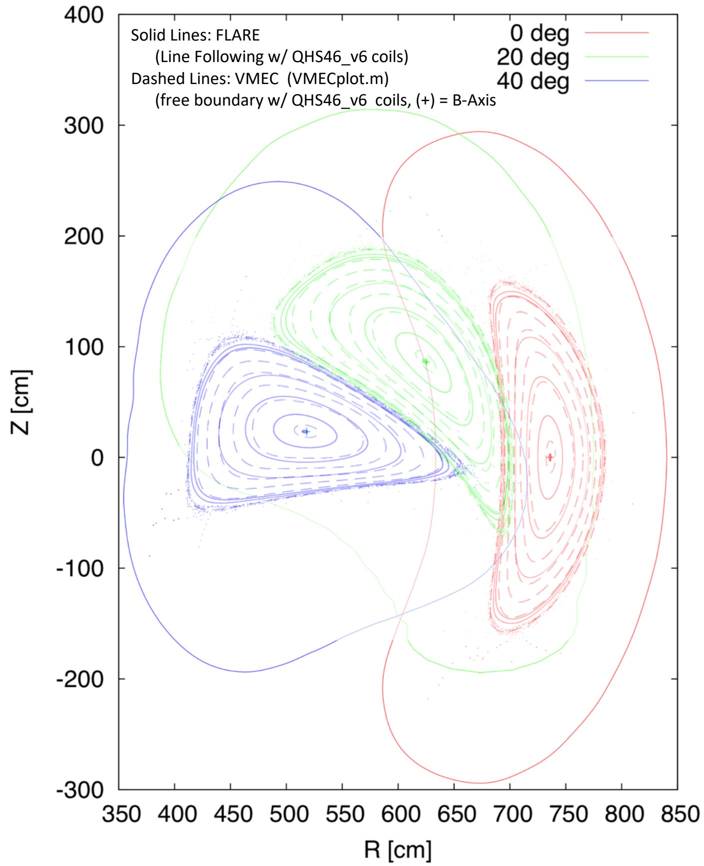
Poincaré plot with field line following compared to VMEC surfaces show agreement up to rational surface at edge

- VMEC vacuum surfaces agree well with field line following (by eye)
  - Axis is close
  - Shapes are qualitatively similar
  - VMEC's 'last closed flux surface' encloses a small stochastic region
    - Not unexpected





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Comparison of QHS46 Fourier Optimized coils to HSX coils which have been scaled to the same effective major radius as QHS46

- The (HSX) QHS configuration and coilset are scaled to match the same effective major radius as the QHS46 configuration that I have been using for the STELLOPT/REGCOIL studies
  - Scale factor: 6.025006 / 1.211364 (~ 5)
- The scaled QHS configuration was run through VMEC (fixed boundary)
  - Started with input.QHS_Rstart_1_513_32polmodes_18x24_axis_v2
  - Scaled the boundary. New filename: input.hsx_scaled_to_QHS46
    - I forgot to scale the axis; VMEC detected this, found a new axis and proceeded to converge
  - Started with coils.hsx_1fil
  - Scaled the coils (with read_coils_scaled.m, a modified version of read_coils.m)
  - Coil data structure is stored in matlab data file: HSX_coils_scaled_to_QHS.mat

- Slide 29: Short list of geometric quantities for the two configurations
- Slides 30-31: |B| on the LCFS and a subset of the coils for this scaled configuration are compared to the |B| and coils of QSH46 with the Fourier Optimized Winding Surface (FOWS)
- See additional slides: The individual coils are plotted on the same axes

Slide 32 – Final comments

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A comparison of a few geometric quantities (from the threed1 output files)

HSX, scaled

- **Aspect Ratio = 9.970152**
- Mean Elongation = 3.851684
- **Plasma Volume = 43.430845 [M**3]**
- **Cross Sectional Area = 1.147258 [M**2]**
- Normal Surface Area = 193.205616 [M**2]
- Poloidal Circumference= 4.693414 [M]
- **Major Radius = 6.025005 [M] (from Volume and Cross Section)**
- **Minor Radius = 0.604304 [M] (from Cross Section)**
- Minimum (inboard) R = 4.438649 [M]
- Maximum (outboard) R = 7.525447 [M]
- Maximum height Z = 1.562752 [M]

QHS46, 6 Coils on FOWS

- **Aspect Ratio = 6.699959**
- Mean Elongation = 4.064947
- **Plasma Volume = 96.174025 [M**3]**
- **Cross Sectional Area = 2.540507 [M**2]**
- Normal Surface Area = 293.721616 [M**2]
- Poloidal Circumference= 7.033194 [M]
- **Major Radius = 6.025006 [M] (from Volume and Cross Section)**
- **Minor Radius = 0.899260 [M] (from Cross Section)**
- Minimum (inboard) R = 4.108455 [M]
- Maximum (outboard) R = 7.840492 [M]
- Maximum height Z = 1.951591 [M]

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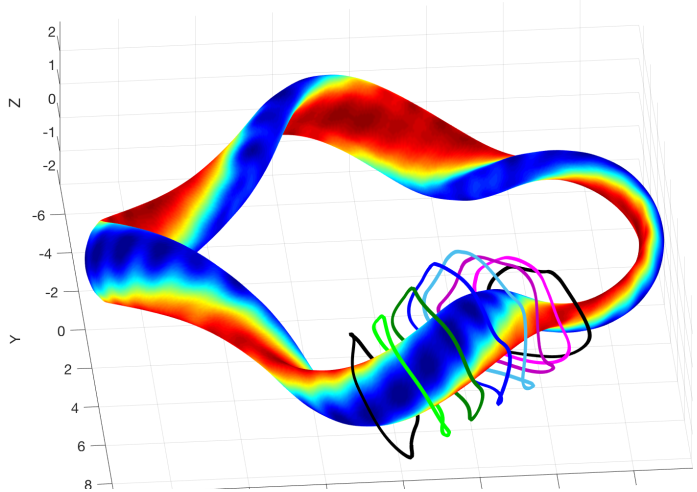
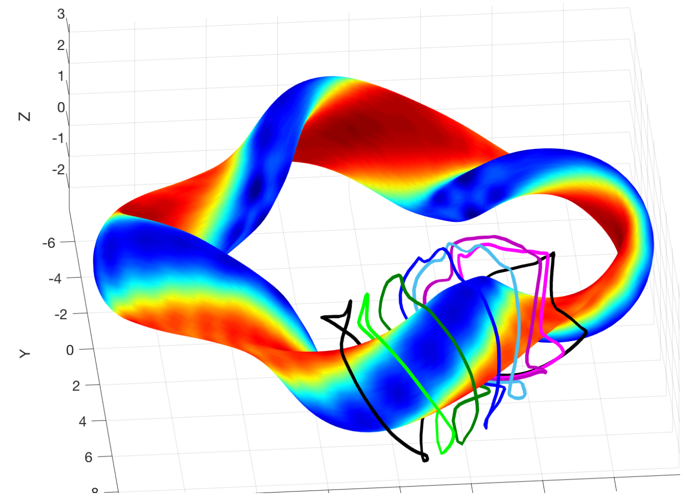
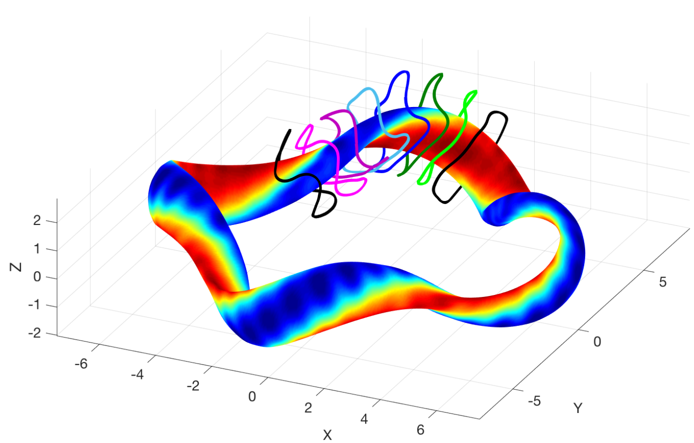
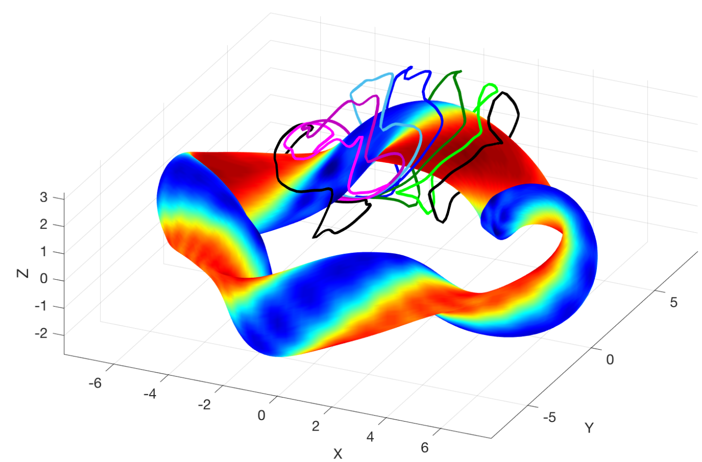
Left: HSX (scaled) Right: QHS46

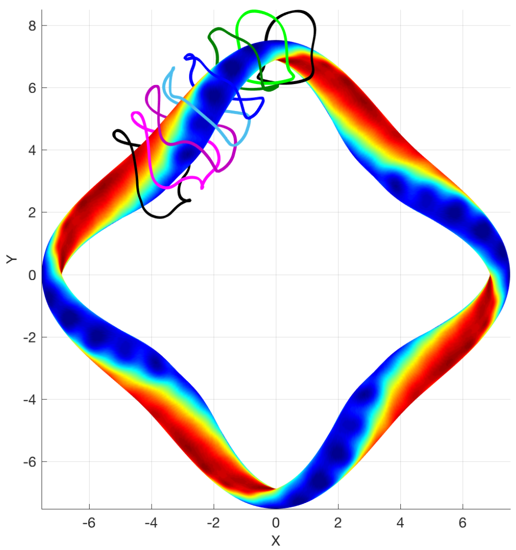


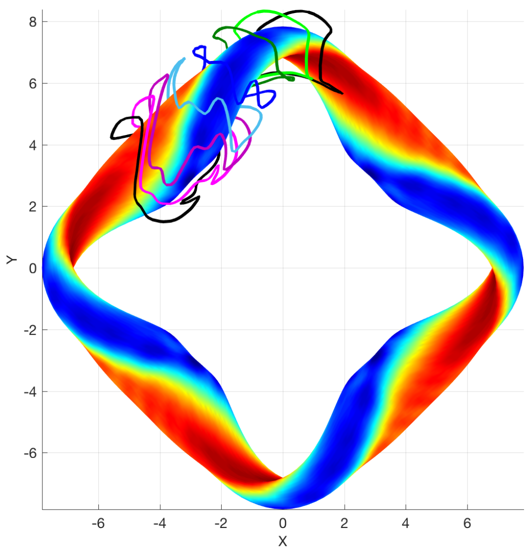






Left: HSX (scaled) Right: QHS46





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Final comments

- Visual inspection shows that the coils based on the Fourier Optimized Winding Surface (FOWS) are qualitatively similar to the HSX coils
  - Longer and more complicated winding path
  - Twice the plasma volume (96 m^3 vs 43 m^3) and smaller aspect ratio (6.7 vs. 10.0).
  - As one distorts the FOWS coils towards the scaled HSX coils, the plasma volume shrinks and the spectrum changes.
- Loose ends, next steps
  - The initial runs have some small 'buggy' constraints. Will re-run to confirm above results (cluster upgrades on-going)
  - It has been challenging to get VMEC to converge with enough spectral (NTOR, MPOL) components to absolutely confirm the Boozer spectrum comparison. Not unexpected given the pathological boundary constraint
  - High-fidelity Boozer spectrum calculations via field-line following are underway.
  - 'Control' coils (saddle type) to target higher order poloidal modes for 'edge control' (such as through shear) would be interesting to insert at this point

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