Physics Design of Stellarators

A. Bader[1], B.J. Faber[1], M. Drevlak[2], C.C. Hegna[1], H. Frerichs[1], J.C. Schmitt[3]

1)University of Wisconsin-Madison
2) IPP-Greifswald 3) Auburn University

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3D Physics in Tokamaks and Stellarators
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WISCONSIN
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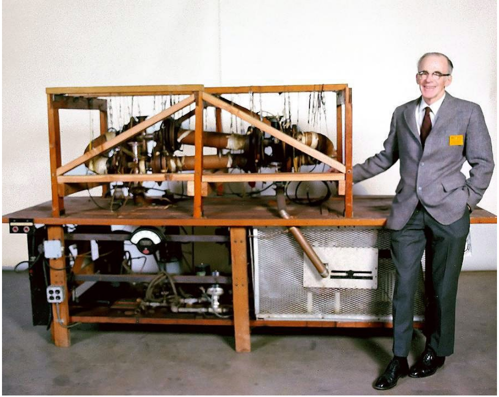
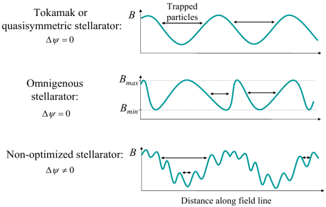
1. How do you design an optimized stellarator?
2. Designing a stellarator: Optimization in practice
   - Neo-classical optimization
   - Energetic particle optimization - testing a metric
   - Optimizing for turbulence - designing a metric
   - Coils - Closing the loop

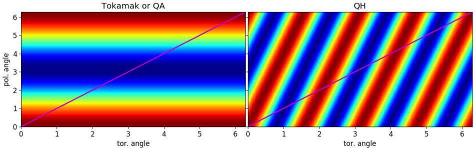
Stellarators are one of the earliest fusion concepts



- Lyman Spitzer invented the stellarator concept in 1953
- Early stellarators suffered from large neo-classical losses
  - Trapped particles precess
  - Axisymmetry: precession is toroidal
  - Non-axisymmetry: precession can have a radial component

Good confinement is attainable in non-axisymmetric systems





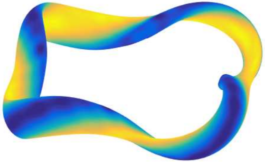
M. Landreman APS invited talk 2012

- If all maxima and minima of $|\mathbf{B}|$ align when following a field line, bounce averaged radial drift is zero
- If $|\mathbf{B}|$ along field is close to sinusoidal, the configuration is quasi-symmetric (or symmetric)

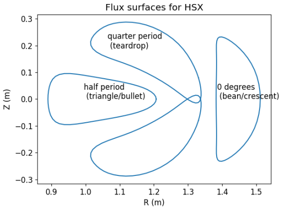
Equilibria are defined by boundaries, $p(\psi)$, and $J(\psi)$

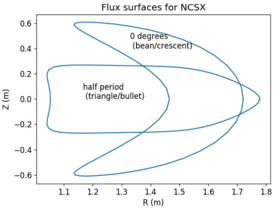
$$R(\theta,\zeta) = \sum_{m,n} R_{c,mn} \cos(m\theta - n\zeta) + R_{s,mn} \sin(m\theta - n\zeta)$$

$$Z(\theta,\zeta) = \sum_{m,n} Z_{c,mn} \cos(m\theta - n\zeta) + Z_{s,mn} \sin(m\theta - n\zeta)$$



- Boundaries given in Fourier series
- Optimized stellarators typically go from bean-like cross-sections to triangle-like cross-sections

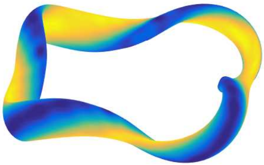




Equilibria are defined by boundaries, $p(\psi)$, and $J(\psi)$

$$R(\theta,\zeta) = \sum_{m,n} R_{c,mn}\cos(m\theta - n\zeta) + \cancelto{0}{R_{s,mn}\sin(m\theta - n\zeta)}$$

$$Z(\theta,\zeta) = \sum_{m,n} \cancelto{0}{Z_{c,mn}\cos(m\theta - n\zeta)} + Z_{s,mn}\sin(m\theta - n\zeta)$$



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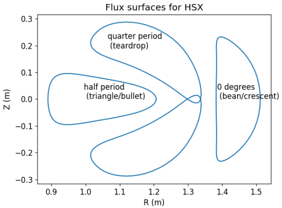
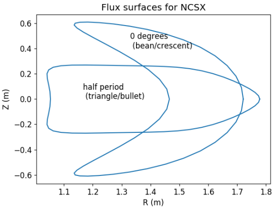
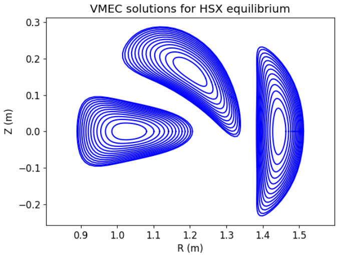
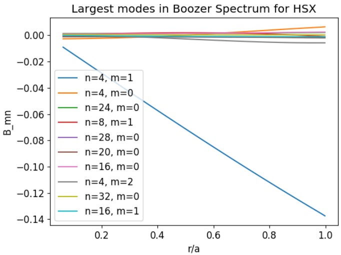




3D geometry allows optimization in novel ways

- Boundary → Eq. solution → Coord. transformations → evaluation





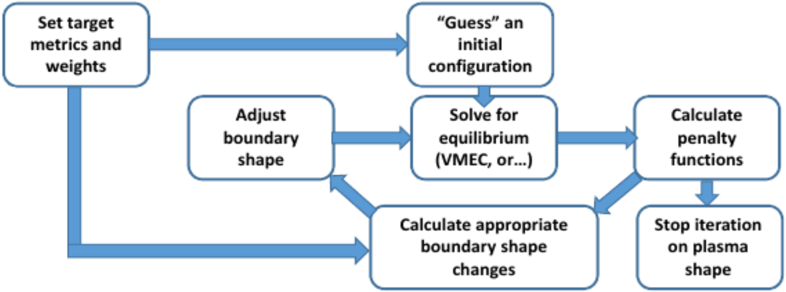
Standard metrics

- Rotational transform, $\iota(\psi)$
- Quasisymmetry metric, $Q(\psi)$
- Neo-classical transport, $\epsilon(\psi)$, $\chi \sim \epsilon(\psi)$
- Bootstrap current, $J_{bs}(\psi)$
- Magnetic well, aspect ratio, volume

New metrics

- Energetic particle confinement
- Turbulent transport
- Coil properties

Optimizing the boundary with modern computation



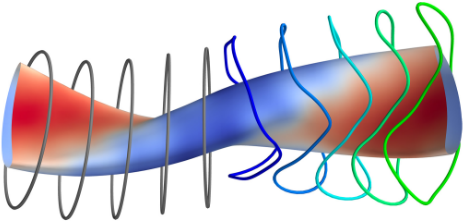
- Optimization codes include `STELLOPT` and `ROSE`
- Usually optimization schemes are modified gradient descents

Optimizer evaluates performance based on user selected penalty functions, p_i, targets t_i and weights w_i

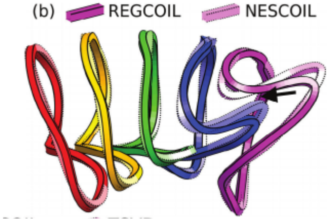
$$F\left(\{R_{mn}, Z_{mn}\}\right) = \sum_i w_i \left[p_i\left(\{R_{mn}, Z_{mn}\}\right) - t_i\right]^2$$

Producing the configuration with coils

Typically coil design is done after an equilibrium is found
This requires iterations between the plasma equilibrium and a coil code to find an adequate solution for both.



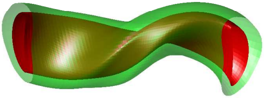
C. Zhu NF **58** (2017)



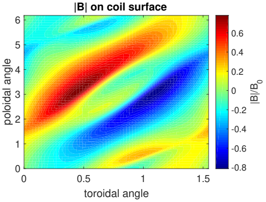
M. Landreman NF **57** (2017)

Standard approach - solve for current potential on external surface

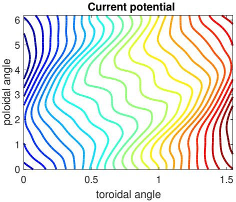
Start with target and coil surfaces



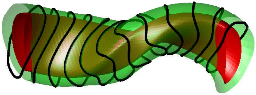
Calculate $|B|$ on coil surface



Integrate to get current potential



Map current potential back onto coil surface



- This approach was used to generate coils for W7-X and HSX
- Modern computational power allows for improvements

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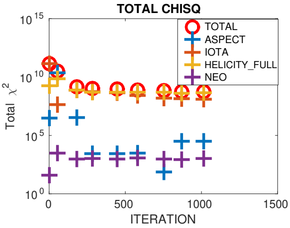
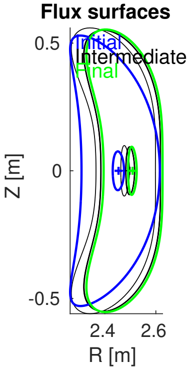
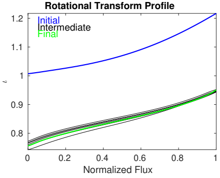
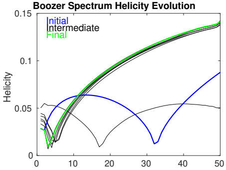
Example - optimize to alter rotational transform profile

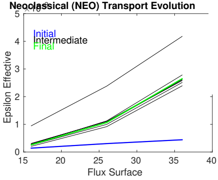


- Major focus is on ι profile at fixed aspect ratio
- However, try to keep good QS and neoclassical transport









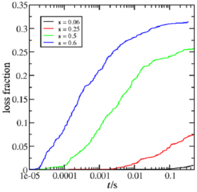
Energetic particle confinement is a key issue in stellarators

- Prompt alpha particle losses can cause significant damage to first wall
- Example - ARIES reactor study found 5% alpha energy losses
- Alpha losses alone exceeded wall heat flux limits at several places
- Evaluation of energetic particle confinement is usually done with Monte-Carlo
  - Computationally expensive
  - Obscures physics mechanisms

Mau FST 2008

Possible metrics for energetic particle optimization

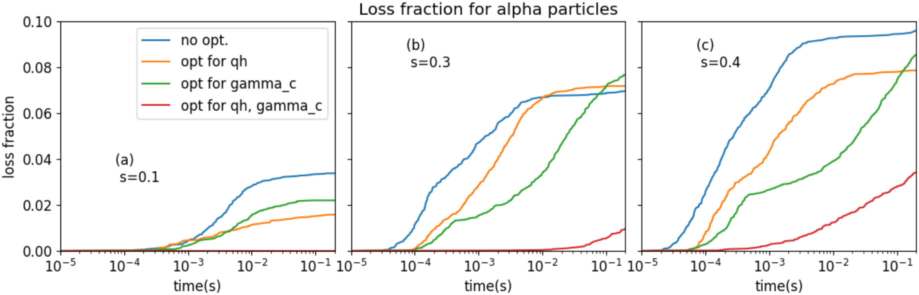
- ϵ_{eff}: standard optimization for neo-classical transport
  - Focuses on deeply trapped particles
  - Less effect on particles near trapped-passing boundary
- Quasi-symmmetry
  - Perfect quasi-symmetry has no particle losses
  - Perfect quasi-symmetry is not actually attainable
- $\gamma_c = \arctan\left(v_r/v_\theta\right)$
  - Seeks to reduce ratio of radial to poloidal drift by aligning J contours
  - Successful at optimizing QH



Henneberg NF 2019

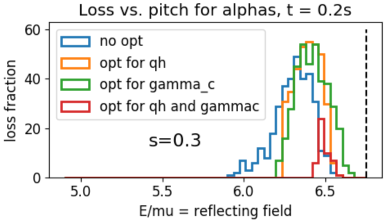
Nemov PoP 1999, Spong PoP 1998, Nemov PoP 2008

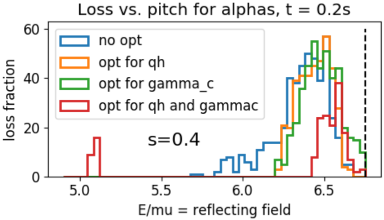
Best performance when optimization for Γ_c and quasisymmetry



- All configurations scaled to 450 m^3 and B_0 = 5.6 T
- $\Gamma_c \sim \sum_{E/\mu} \sum_{\mathrm{wells}} \int_b \arctan^2\left(v_r/v_\theta\right)\tau_b$
- Prompt losses entirely eliminated in best performing case (red); all losses eliminated inside s = 0.1

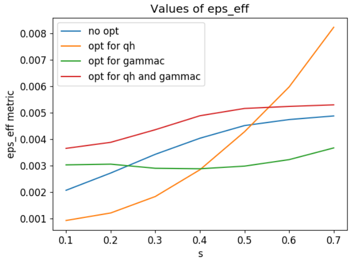
Particle losses at trapped passing boundary are suppressed

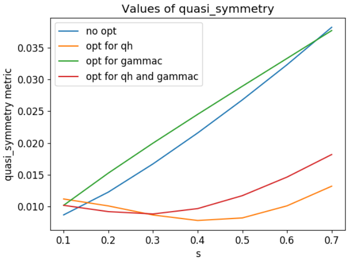


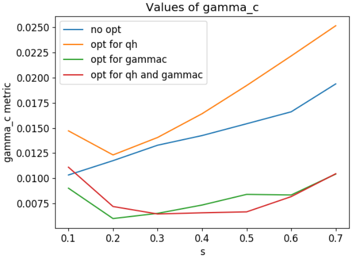


- Most problematic region is near the trapped passing boundary
- The best confinement case (red) sacrifices confinement of deeply trapped particles to better confine particles near the trapped passing boundary

ϵ_{eff} is not the correct metric for energetic particles







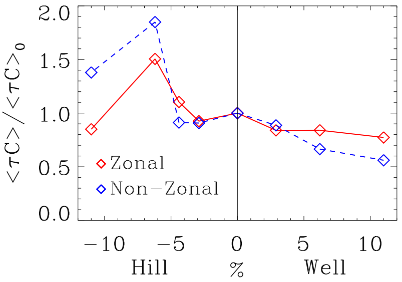
- Improvement of alpha confinement in red configuration despite worse ϵ_{eff} across most of the radius

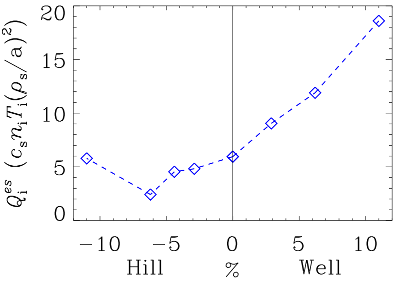
Using energy transfer as a turbulent transport metric

- Stable modes can provide an energy sink at instability scales
- Stable modes couple to linear instability through 3-wave nonlinearity.
  - Heat flux is inversely proportional to a correlation time, τ
  - $Q \propto \sum_{i,j,k} \frac{1}{k_\perp^2} \frac{\gamma_k}{\tau}, \ \tau_{ijk} = 1/i\left(\omega_i^* - \omega_j - \omega_k\right)$
  - Key idea: maximize correlation lifetimes between unstable and stable modes
- Geometry and plasma profiles determine the ω values

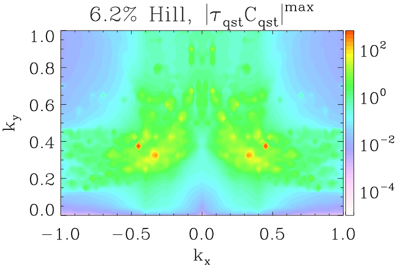
C.C. Hegna PoP **25** (2018) 022511

Turbulence metric reproduces non-linear gyrokinetics



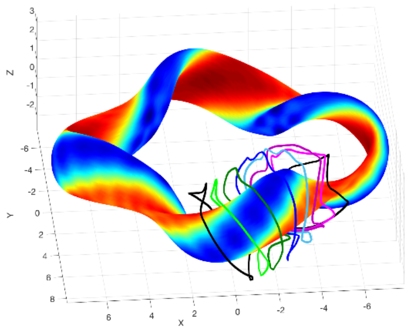


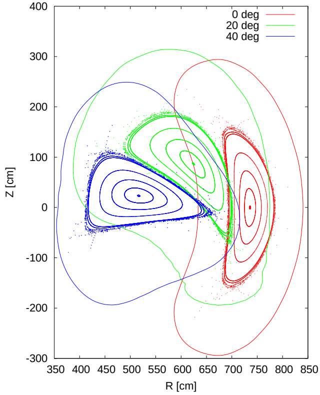
- Metric anti-correlated with gyrokinetic Q as predicted
- Resonances of $1/\tau$ at specific k_x, k_y cause improved performance



Incorporating coil codes directly into equilibrium optimization

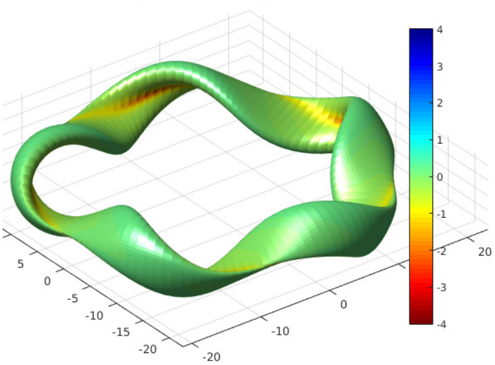
- Increases search space considerably
- May require better optimization algorithms

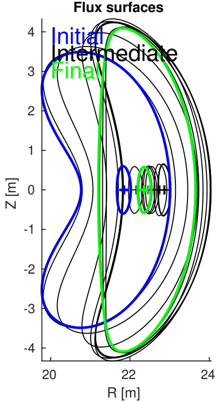




Designing metrics for ease of coil design

- Sensitive locations for coils correspond to areas of high second principal curvature of the boundary
- These areas can be directly targeted in optimization codes



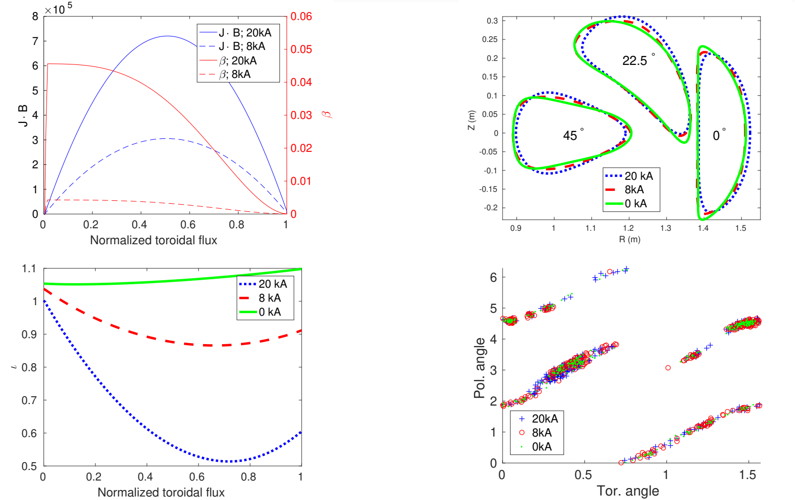


We are ready to design the next generation of optimized stellarator

- Stellarator optimization has been successfully implemented on HSX and W7-X
  - Optimizer codes manipulate plasma boundaries and evaluate the resulting equilibria
  - Separate codes determine coil shapes
- Using 3D nature of stellarators we can optimize configurations in completely new ways
  - New metrics for energetic particle confinement and turbulent transport are being tested and developed
  - Coil metrics or coil design itself can be included in the physics optimization
  - Development of better optimization algorithms and boundary representations is ongoing

Extra slides

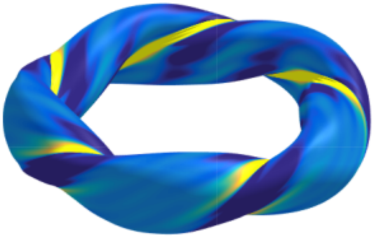
Non-resonant divertors resilient to plasma evolution

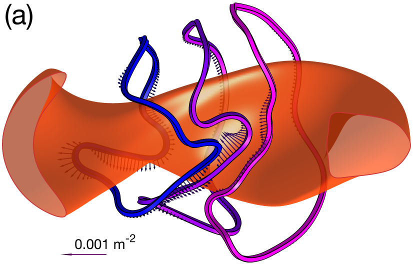


No difference is noticeable with shape changes from plasma!

A. Bader PoP **24** (2017) 032506

Coil sensitivity and divertor locations





- Shape gradient calculations indicate regions of high sensitivity
- Sensitive regions are far away from desirable divertor locations

E.J. Paul NF **58** (2018) 076015; M. Landreman NF **58** (2018) 076023