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Pitfalls and Paths Forward for Stellarator Optimization

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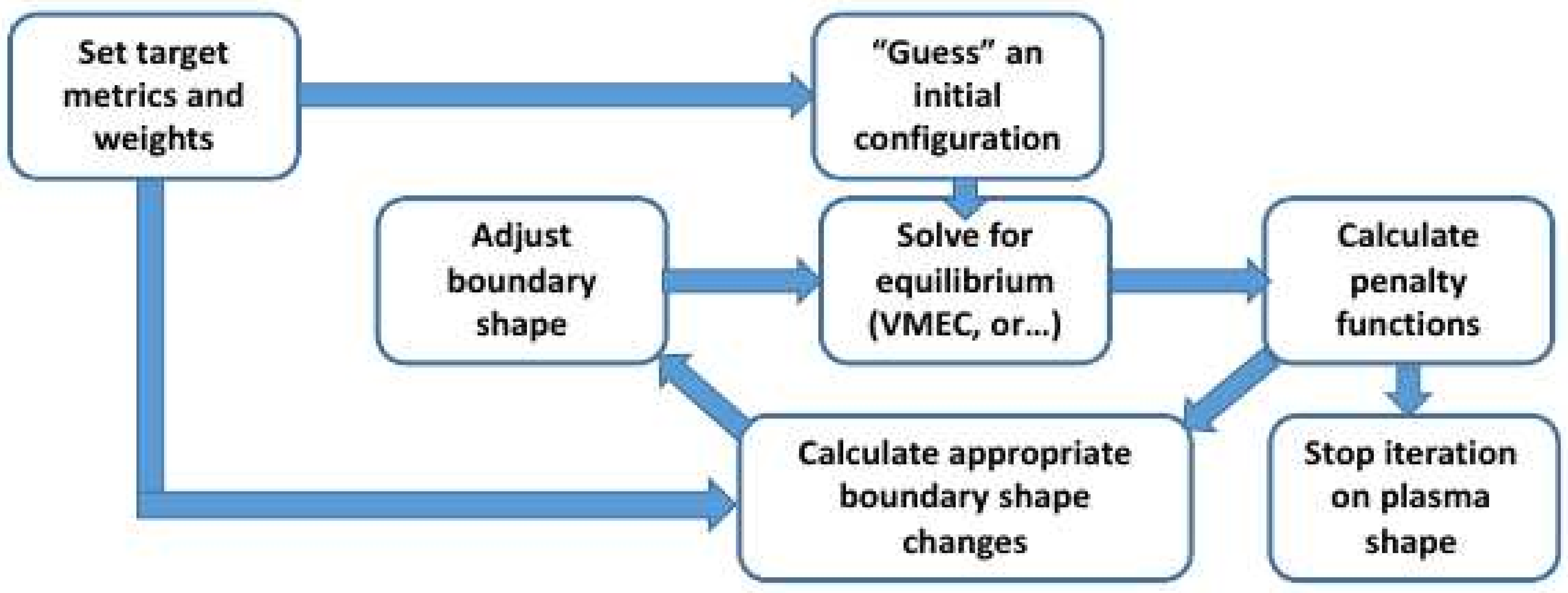
Optimization in Stellarators

- Configuration optimization focuses on manipulating plasma boundary



- Boundary given as Fourier series

$$R(\theta,\zeta) = \sum_{m,n} R_{mn}\cos(m\theta - n\zeta)\,;\; Z(\theta,\zeta) = \sum_{m,n} Z_{mn}\sin(m\theta - n\zeta)$$

- Fourier mode representation convenient for equilibrium codes
- R_{mn}, Z_{mn} are independent variables, typically on order ~100 are used
- With equilibria solutions, penalty functions (p_i) are calculated and compared to targets (t_i) with weights (w_i)

$$F(\{R_{mn}, Z_{mn}\}) = \sum_i w_i (p_i - t_i)^2$$

- Optimization codes include **STELLOPT**[1] and **ROSE**[2]

- Any quantity calculable from an equilibrium can be optimized
  - Rotational transform profile: $\iota(\psi)$
  - Quasi symmetric metric: $Q(\psi)$

$$Q_{qa} = \sum_{n\neq 0} B_{mn}^2/B_{00}^2;\; B(\psi,\theta,\zeta) = \sum_{m,n} B_{\psi,m,n}\cos(m\theta - n\zeta)$$

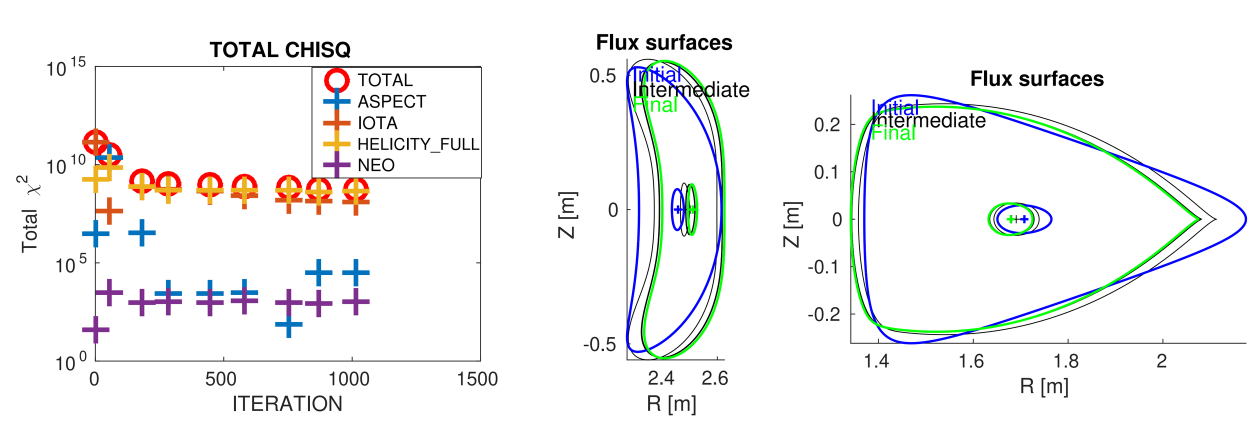
  is the target for quasi-axisymmetry
  - Neo-classical transport: $\epsilon_{\text{eff}}(\psi)$
  - Other quantities include: magnetic well, stability considerations, aspect ratio, alpha confinement, turbulent transport (see Hegna talk) etc.

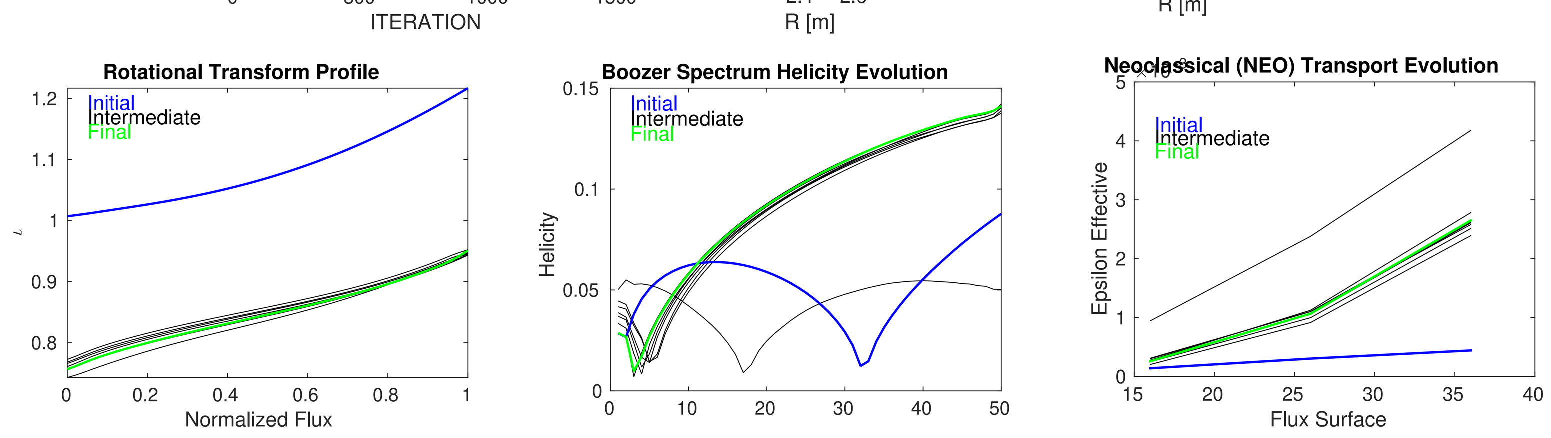
- Coil optimization is usually separate from configuration optimization
  - Machine design requires iteration between coil codes and configuration optimization codes
  - Integration of coils into optimizers complicates problems

Optimization Algorithms

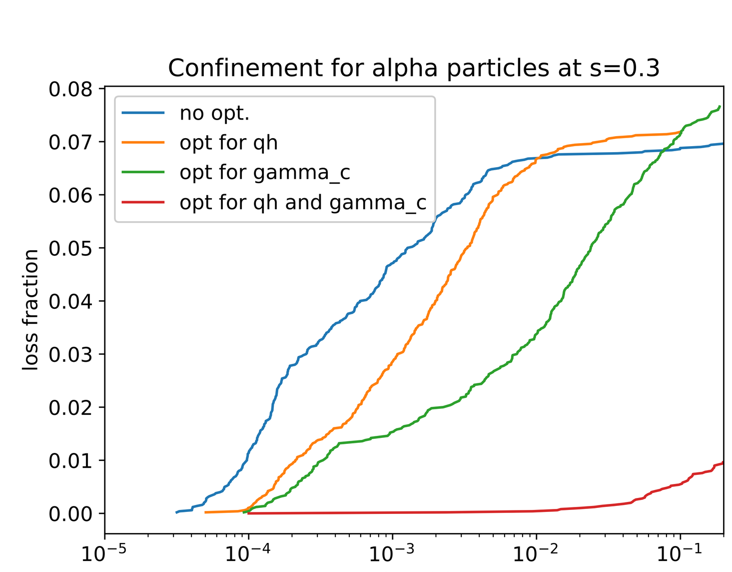
- Optimizations typically use gradient descent methods
- Calculate $\partial p_i/\partial x_j$, where $x_j \in \{R_{mn}, Z_{mn}\}$
- Common methods: Levenberg-Marquardt, Brents, Quasi-Newton

Optimization for ι, quasi-helical symmetry, neoclassical transport, and aspect ratio using Levenberg-Marquardt algorithm in **STELLOPT**





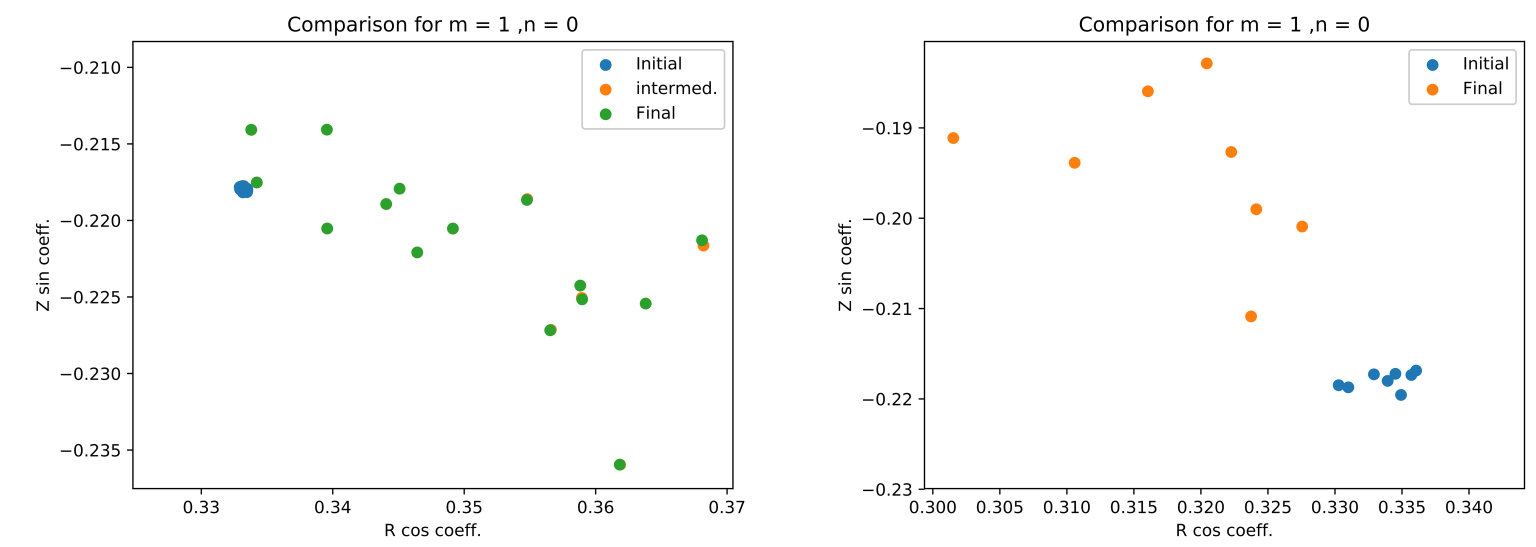
- Example: optimization for energetic particle confinement
- Optimizer finds solution with low energetic particle losses
- Best case (red): optimizer successfully improves both quasi-symmetry and Γ_c, an energetic particle metric
- Results: losses greatly reduced near trapped-passing boundary



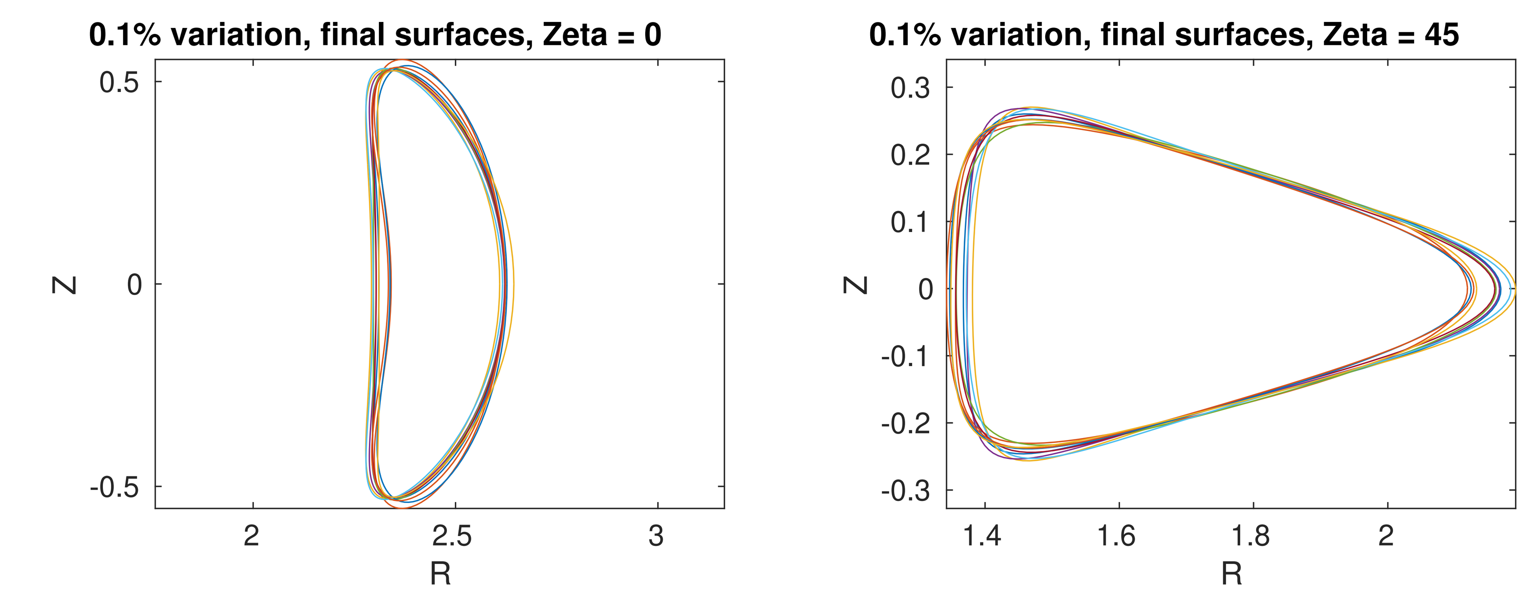
Pitfalls

- Search space is non-Convex
  - Initialize with small differences in starting coefficients but exact same targets and weights
  - Final solutions wind up in different places
  - Local minima abound

STELLOPT **ROSE**

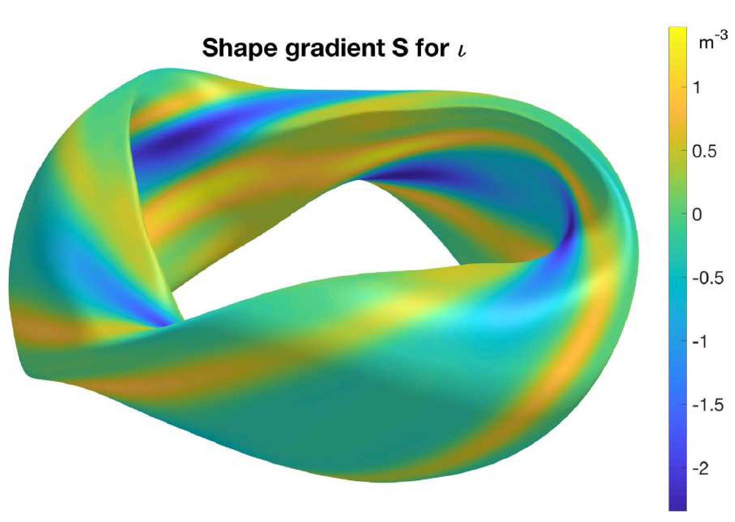


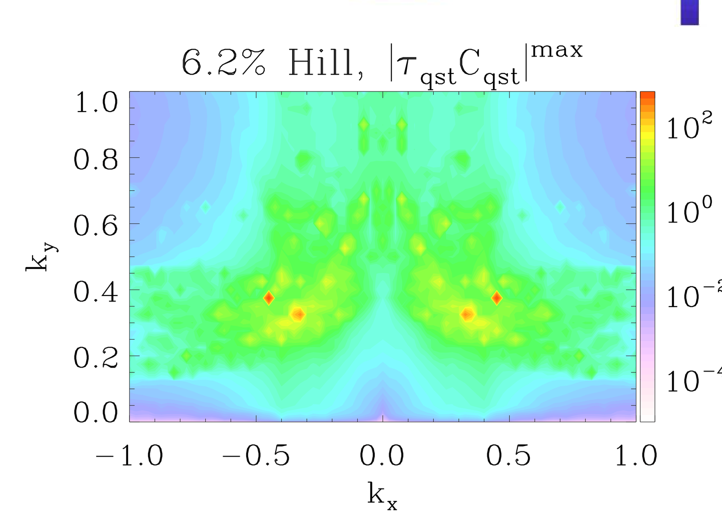
Initial and final values of $R_{m=1,n=0}$ and $Z_{m=1,n=0}$ modes when all coefficients are varied by a small random value less than 1%



- Final configurations are different
- Our optimization algorithms do not find global minima

- Variables for optimization (Fourier modes) are global parameters
- Fourier coefficients obscure effects of boundary deformations
- Shape gradients can often inform the desired locations for variations [3]
- Local variations are desirable
- Some metrics (turbulent coupling coefficients) attempt to optimize resonant features
- Landscape is dotted by local minima, how to find the best one?





Paths Forward

- Improve optimizers
  - More robust routines, resistant to local minima
  - Recipes for improved optimization - how can we use our tools better?
- Local boundary representations
  - Move away from Fourier modes for boundary representation
  - Possibility: local 2D splines fit to boundary points
  - New algorithms to solve for equilibria (replacement for VMEC)
  - Alternative - local equilibrium calculations
  - Alternative - vary points on boundary and then fit to a Fourier series with arbitrary precision
- SIMONS collaboration on improving algorithms and boundary representations is greatly appreciated

References

[1] Lazerson, S.A. STELLOPT Computer Software

[2] Drevlak, M. Nuc. Fus. **59** (2019)

[3] Landreman, M. Nuc. Fus. **58** (2018)

Acknowledgments

Work funded by UW2020 135AAD31136 and DE-FG02-93ER54222

SIMONS Hidden Symmetries New York, NY, USA, March 27, 2019