

A NEW METHOD FOR THE OPTIMIZATION OF FINITE BUILD STELLARATOR COILS



L. Singh¹, T. Kruger¹, C. Zhu², S. Hudson², A. Bader¹, D.T. Anderson¹
University of Wisconsin Madison¹, Princeton Plasma Physics Laboratory²



Background

- Existing stellarator coil optimization codes use a zero-thickness single filament model that only approximates finite build coils.
- Finding a coil set with desirable physics and engineering properties is a crucial step in the design of new stellarators.
- Here we present a new method for the optimization of finite build. We show initial results for the HSX stellarator and a candidate configuration for a new UW-Madison stellarator.

Objective of Stellarator Coil Optimization

- The goal of all coil optimization codes is to find a coil set that recreates a specified magnetic equilibria. This is achieved by minimizing magnetic field $\mathbf{B}$ error on the last closed flux surface:

$$f_B = \int \left(\frac{\mathbf{B} \cdot \hat{\mathbf{n}}}{B} \right)^2 dS$$

- Feasible coil sets are determined by engineering constraints including coil-coil separation, coil curvature, and plasma accessibility, among others.

The Multifilament Model

- To model finite build coils we build on FOCUS single filament model [c. Zhu et al., Nucl. Fusion, 58, 016008 (2018)].

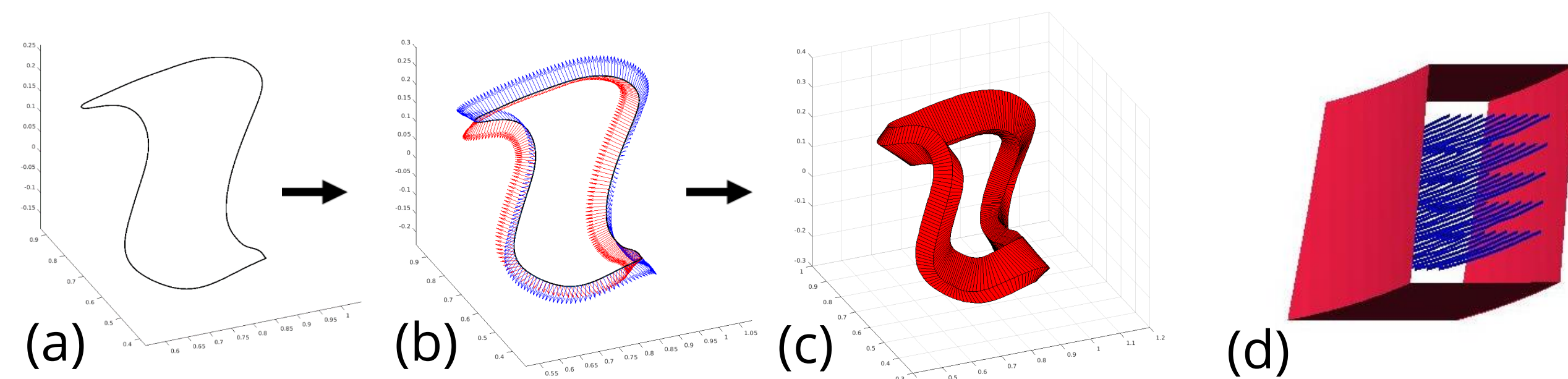


Fig 1. Construction of finite build coils: [a] FOCUS single filament [b] basis vectors for finite build overlaying FOCUS single filament [c] resulting finite build coil. [d] Cross section of finite build coil encasing a multifilament.

- Start by finding position along single filament coil $\mathbf{r}$ using a Fourier series.

$$\mathbf{r}(s) = \left[\sum_{n=0}^N \mathbf{X}_{c,n} \cos(ns) + \mathbf{X}_{s,n} \sin(ns) \right] \quad \mathbf{X}_{c,n} = \begin{pmatrix} x_{c,n} \\ y_{c,n} \\ z_{c,n} \end{pmatrix} \quad \mathbf{X}_{s,n} = \begin{pmatrix} x_{s,n} \\ y_{s,n} \\ z_{s,n} \end{pmatrix} \quad s \in [0, 2\pi]$$

- For each position on coil, calculate basis vectors $\hat{\mathbf{t}}$, $\hat{\mathbf{n}}$, $\hat{\mathbf{b}}$ via differentiation and the Gram-Schmidt process, using both position vectors $\mathbf{r}$ and coil centroids $\mathbf{X}_c$:

$$\mathbf{x}(s) = \mathbf{r}(s) - \mathbf{X}_c \quad \hat{\mathbf{t}} = \frac{\partial \mathbf{r} / \partial s}{|\partial \mathbf{r} / \partial s|} \quad \hat{\mathbf{n}} = \mathbf{x} - (\mathbf{x} \cdot \hat{\mathbf{t}}) \hat{\mathbf{t}} \quad \hat{\mathbf{b}} = \hat{\mathbf{t}} \times \hat{\mathbf{n}}$$

- Replace single filament with a multifilament according to user-defined coil cross section dimensions, which specify lengths along $\hat{\mathbf{n}}$ and $\hat{\mathbf{b}}$ directions.

Optimization of Coil Rotation

- Winding pack rotation is determined by an angle function α , which is parameterized using a Fourier series. This embeds flexibility to optimize the rotation profile of finite build coils.
- Basis vectors are rotated by angle α about $\hat{\mathbf{t}}$, causing coil cross section to rotate.
- A steepest descent forward tracking algorithm is used to optimize Fourier components $\alpha_{c,n}$ and $\alpha_{s,n}$ for each coil.

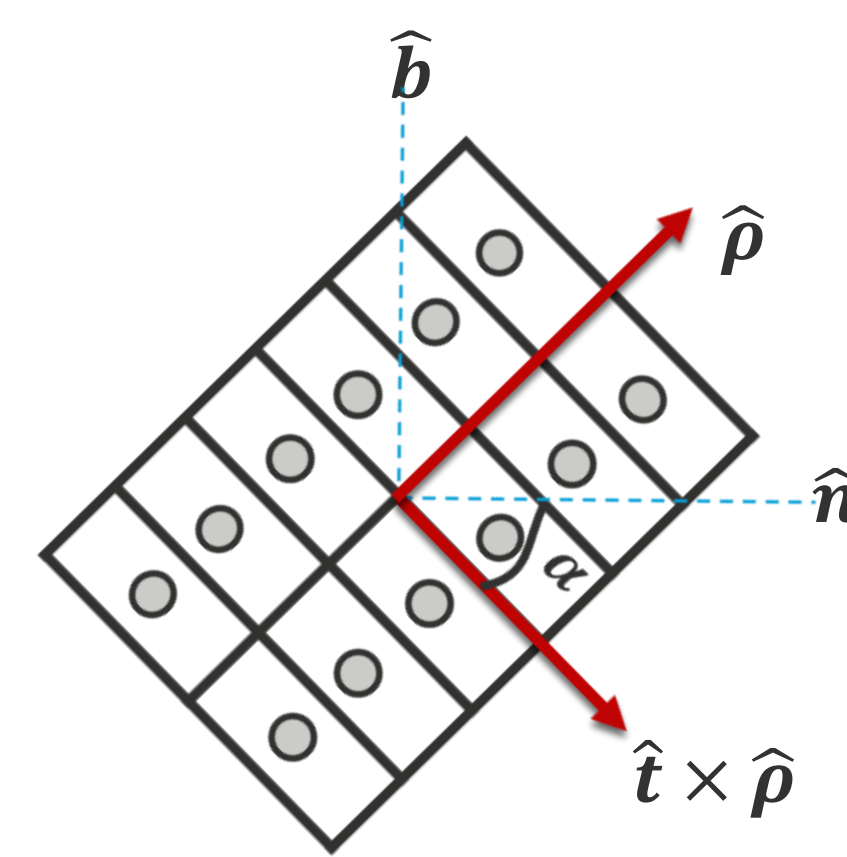


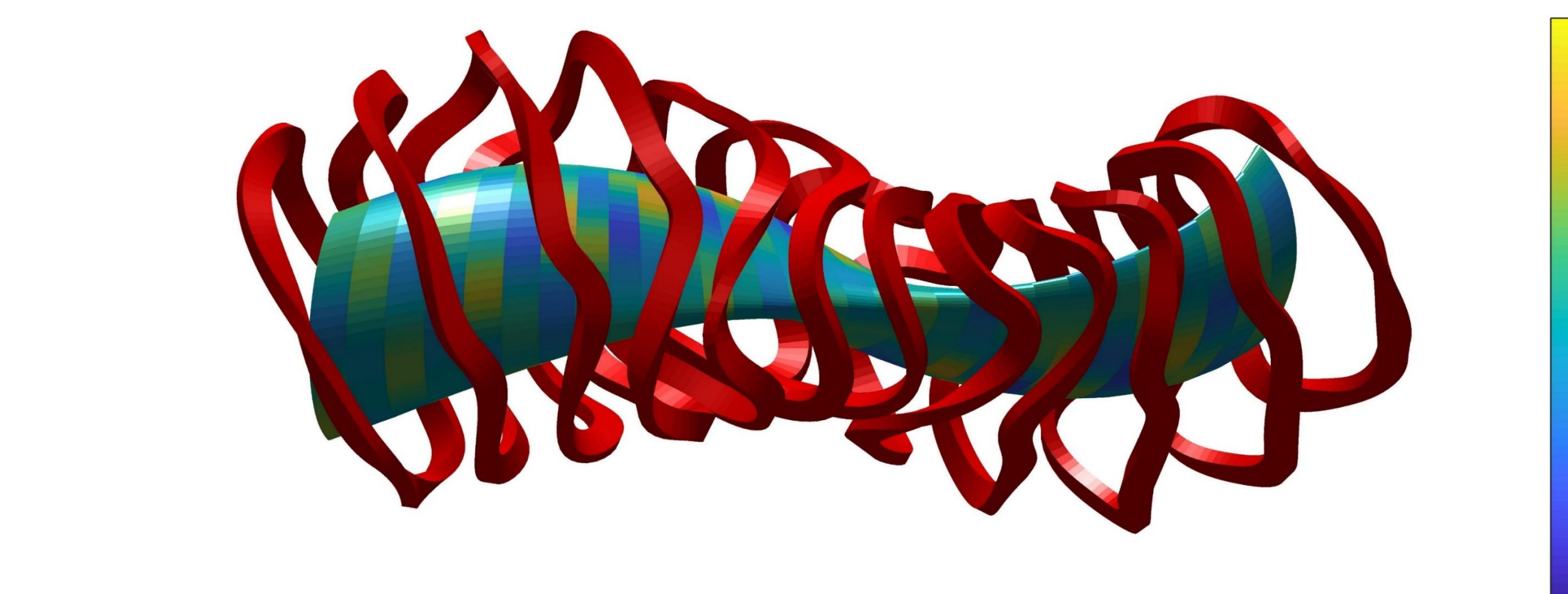
Fig 2. Rotation of coil cross section by angle alpha

$$\alpha(s) = \sum_{n=0}^N \alpha_{c,n} \cos(ns) + \alpha_{s,n} \sin(ns) \quad s \in [0, 2\pi]$$

$$\hat{\rho} = \cos(\alpha) \hat{\mathbf{n}} + \sin(\alpha) \hat{\mathbf{b}}$$

$$\hat{\mathbf{t}} \times \hat{\rho} = -\sin(\alpha) \hat{\mathbf{n}} + \cos(\alpha) \hat{\mathbf{b}}$$

Application: HSX Stellarator



Alpha harmonics per coil: 2
Build dimension: 6cm. x 3cm.
Multifilament packing: 4x3 grid

Single filament rms error: 6.0930 x 10⁻³
Unoptimized multifilament rms error: 6.9681 x 10⁻³
Optimized multifilament rms error: 6.1257 x 10⁻³
Reduction in rms error after optimization: 12.1%

Single filament mean coil-plasma distance: 14.57 cm.
Single filament min. coil-plasma distance: 14.37 cm

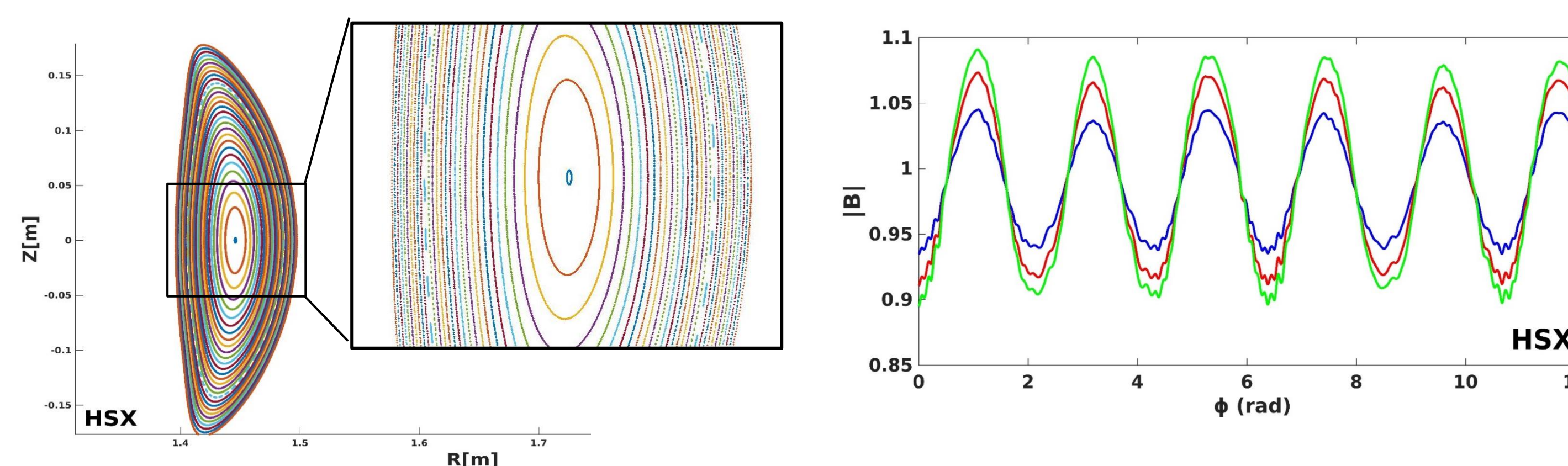
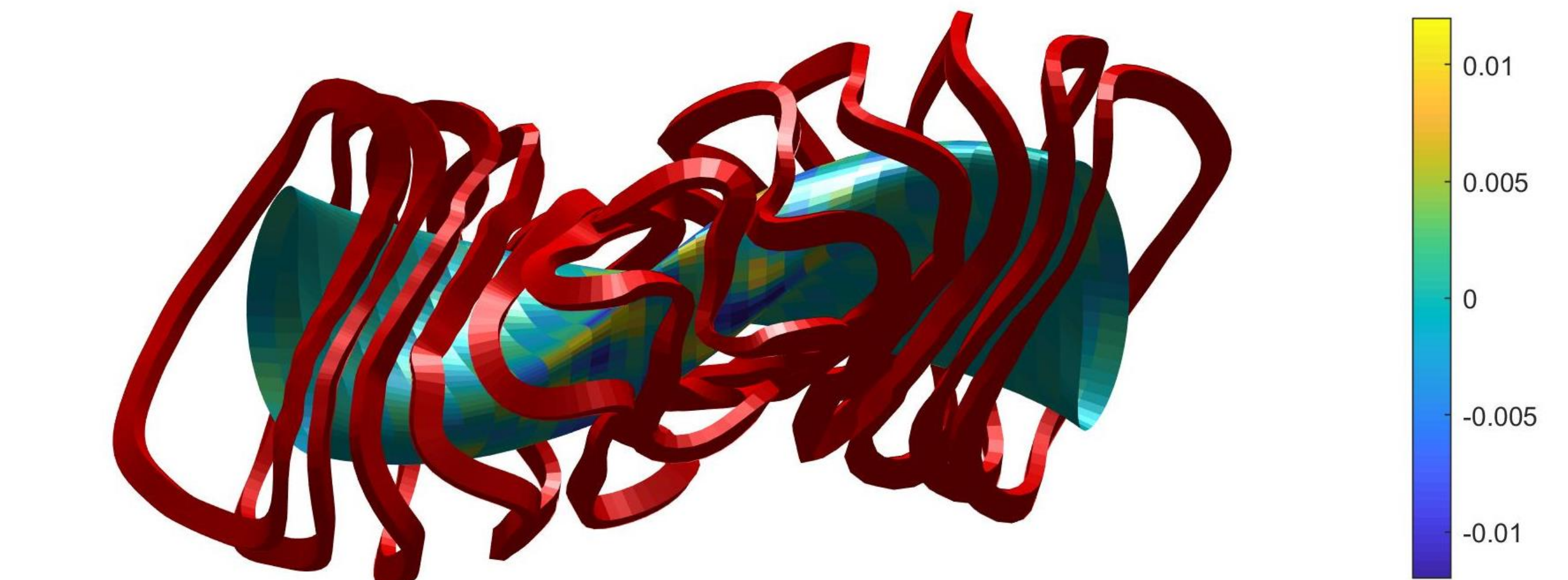


Fig 3. Coils for the HSX stellarator: Shown are the finite build coils and the single filament coils. The color bar on the finite build figure indicates extent of magnetic field error on the last closed flux surface. Also shown are flux surfaces for the optimized multifilament plasma and $|\mathbf{B}|$ along field lines at three different r/a : 1/3, 1/2, and 2/3.

Application: New UW QHS Configuration



Alpha harmonics per coil: 2
Build dimension: 12cm. x 6cm.
Multifilament packing: 4x3 grid

Single filament rms error: 5.3261 x 10⁻³
Unoptimized multifilament rms error: 7.4424 x 10⁻³
Optimized multifilament rms error: 6.1257 x 10⁻³
Reduction in rms error after optimization: 17.7%

Single filament mean coil-plasma distance: 19.50 cm.
Single filament min. coil-plasma distance: 22.49 cm

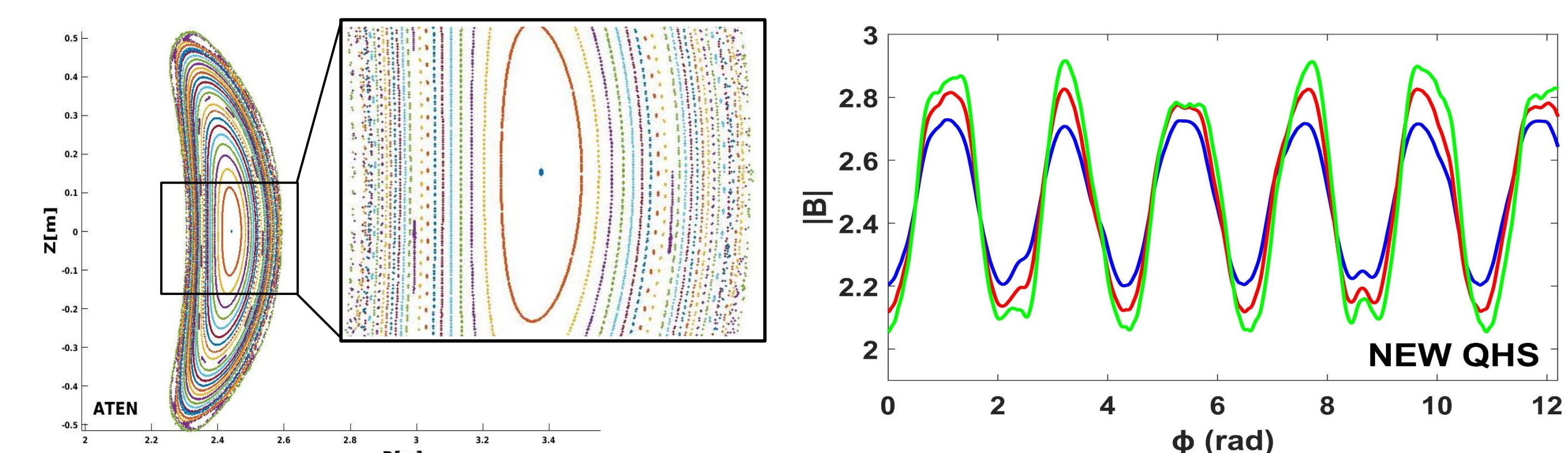


Fig 4. Coils for a candidate QHS design: Shown are the finite build coils and the single filament coils. The color bar on the finite build figure indicates extent of magnetic field error on the last closed flux surface. Also shown are flux surfaces for the optimized multifilament plasma and $|\mathbf{B}|$ along field lines at three different r/a : 1/3, 1/2, and 2/3.

Conclusions

- Finite build coil models should be implemented in stellarator coil design, as finite build rotation can significantly affect the confining magnetic field.
- Finite build stellarator coils are capable of recreating magnetic fields with the same fidelity as single filaments.

Future Work

- Implement cost functions for total coil rotation and coil-coil separation
- Optimize finite build centroids and build dimensions
- Implement multi-layer parallelization (OpenMP, MPI, GPU)

This work is supported by the US DOE Contract No. DE-AC02-09CH11466, US DOE Contract No. DE-FG02-93ER54222 and UW2020 Grant 135AAD3116.