Role of 3D Geometry in Reducing Turbulent Transport in Stellarators

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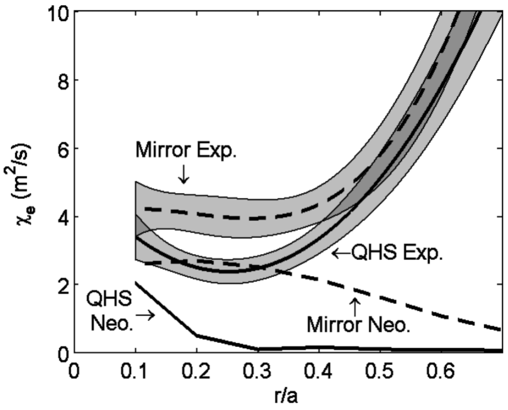
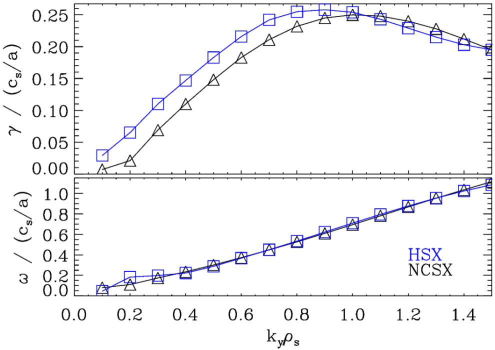
Motivation: reducing turbulent transport

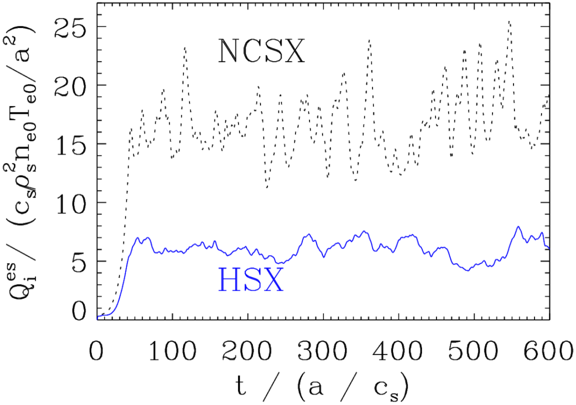
- Stellarator success story: optimizing for neoclassical transport confinement
  - HSX; W7-X $n\tau T$ record triple product
- HSX: remaining transport likely *turbulence* driven
  - See J. Smoniewski 11:00 a.m. Monday
- Optimizing for reduced turbulent transport is a substantial challenge
  - Turbulence in stellarators: both nonlinear and fully 3D
- Can turbulence be approximated effectively by reduced models?



Source: Canik *et al.*, PRL, 2007

Growth rates insufficient to predict stellarator turbulence

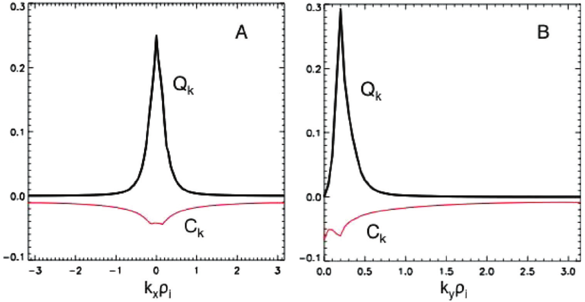


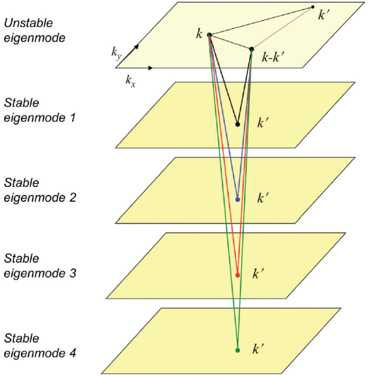


- Similar growth rates observed in ITG simulations for HSX, NCSX; drastically different nonlinear behavior
- See McKinney *et al.*, *J. Plasma Phys.*, 2019 and P.86 on Thursday for more detail

Turbulence saturation through stable modes

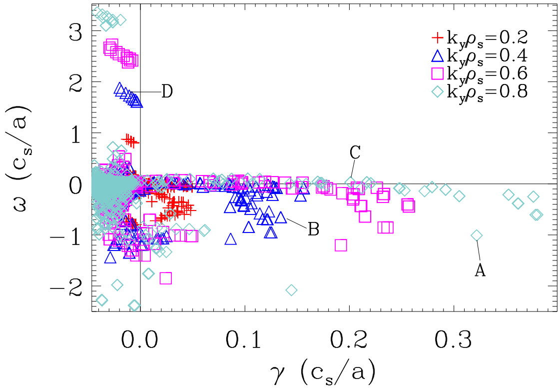
- Nonlinear structures at a given $\mathbf{k}$ can be described by basis of unstable, stable eigenmodes of linear operator
- Nonlinearity transfers energy to stable mode structures
  - Energy sink *at same spatial scale* as instability
- 3D geometry creates new possibilities for turbulence saturation through stable modes

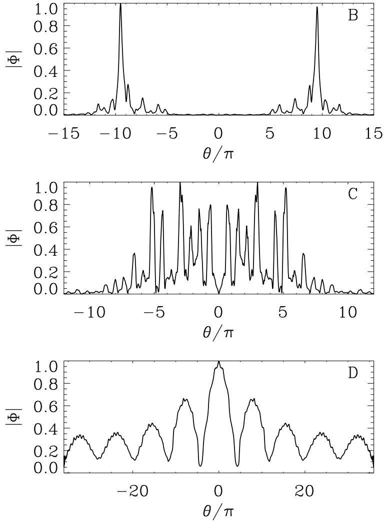




Source: Hatch *et al.*, PoP, 2011

3D geometry – complex drift wave eigenspectrum





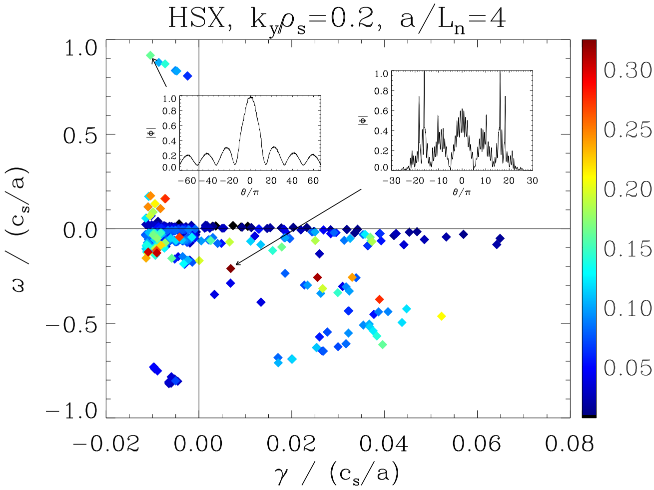
– Subdominant, extended, "slab-like" modes weakly dependent on local curvature in HSX spectrum ⇒ consequences of small global magnetic shear

Signature of stable modes in turbulence

- Project turbulence onto eigenvectors:

$$p_j = \frac{\left| \int \mathbf{dxdv} f_j^* f_{\mathrm{NL}} \right|}{\left(\int \mathbf{dxdv} |f_j|^2 \int \mathbf{dxdv} |f_{\mathrm{NL}}|^2 \right)^{1/2}}$$

- Extended subdominant, stable modes with largest p_j at low k_y
- Suggests stable modes playing prominent role in HSX TEM turbulence saturation



Motivation, Part II

- Gyrokinetic simulations show a complex, geometry-dependent saturation picture
  - Small $\hat{s}$: subdominant modes play an important role in saturation
  - Different from high-$\hat{s}$ tokamaks (zonal flows)
- Major questions:
  - How does stellarator geometry impact turbulence saturation mechanisms?
  - Can 3D geometry be manipulated to impact saturation dynamics to reduce turbulence?
  - Can an improved proxy for turbulent transport be constructed for optimization?
- Explore ITG saturation in 3D geometry with fluid model
  - Provides analytic, tractable saturation theory
  - ITG relevant for next-generation stellarator
  - Apply model to HSX (QHS) and NCSX (QAS) configurations

Fluid model for ITG turbulence saturation in stellarators

- 3-field, nonlinear model for essential ion dynamics: Φ_i, U_i, T_i, and adiabatic e^-

$$\frac{\partial}{\partial t}[\Phi_k + \chi_k(\Phi_k + T_k)] - \mathrm{i}D_k(\Phi_k + T_k) + \frac{1}{\sqrt{g}B}\frac{\partial U_k}{\partial z} = \sum_{\mathbf{k}'}(k_\psi k'_\alpha - k_\alpha k'_\psi)B_{k'k}\Phi_{k-k'}(\Phi_{k'} + T_{k'}),$$

$$\frac{\partial U_k}{\partial t} + \frac{1}{\sqrt{g}B}\frac{\partial}{\partial z}(\Phi_k + T_k) = \sum_{\mathbf{k}'}(k_\psi k'_\alpha - k_\alpha k'_\psi)\Phi_{k-k'}U_{k'},$$

$$\frac{\partial T_k}{\partial t} + \mathrm{i}k_\alpha \epsilon_T \Phi_k = \sum_{\mathbf{k}'}(k_\psi k'_\alpha - k_\alpha k'_\psi)\Phi_{k-k'}T_{k'}$$

- Stellarator geometry encoded by χ_k and D_k terms for $\mathbf{B} = \nabla\psi \times \nabla\alpha$:

$$D_k = (k_\psi \nabla\psi + k_\alpha \nabla\alpha) \cdot \frac{\mathbf{B} \times \nabla B}{B^2},\ \ \chi_k = k_\psi^2 \nabla\psi \cdot \nabla\psi + 2k_\psi k_\alpha \nabla\psi \cdot \nabla\alpha + k_\alpha^2 \nabla\alpha \cdot \nabla\alpha$$

- Eigenmodes given by $\beta_k = (\Phi_k(z), U_k(z), T_k(z))^T$

Three-wave correlation times are important

- Evolution of eigenmode energy determined by triplet correlations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\beta_p(k)\beta_s^*(k)\rangle \propto \mathrm{i}\,(\omega_p - \omega_s^*)\,\langle\beta_p(k)\beta_s^*(k)\rangle + \sum_{k'} C_{k,k'}\left\langle\beta_p(k)\beta_s^*(k)\beta_t(k')\right\rangle,$$

Steady-state: $\Rightarrow \left\langle\beta_p(k)\beta_s^*(k)\beta_t(k')\right\rangle \approx \sum_{k',k''} C_{k,k'}C_{k,k''}\dfrac{\langle\beta_p(k)\beta_s^*(k)\beta_t(k')\beta_q(k'')\rangle}{\mathrm{i}\,(\omega_t + \omega_p - \omega_s^*)}$

- Modifies energy evolution with triplet correlation time τ_{pst} and coupling coefficent $C_{k,k'}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\beta_p(k)\beta_s^*(k)\rangle \propto \mathrm{i}\,(\omega_p - \omega_s^*)\,\langle\beta_p(k)\beta_s^*(k)\rangle + \sum_{k',k''} \tau_{pst}C_{k,k'}C_{k,k''}\langle\beta_p(k)\beta_t^*(k'')\rangle\langle\beta_s(k)\beta_q(k')\rangle,$$

where $\tau_{pst} = 1/\mathrm{i}\,(\hat{\omega}_t + \hat{\omega}_p - \hat{\omega}_s^*)$

Using τ_{pst} as a proxy for turbulent transport

- Express energy evolution as:

$$\frac{d}{dt}\left\langle|\beta_p|^2\right\rangle = \mathrm{i}\left(\omega_p^* - \omega_p\right)\left\langle|\beta_p|^2\right\rangle + \sum_{k',s,t} C_{pst}(k,k')\tau_{pst}(k,k')F(\beta_i,k,k') + C.C.$$

- Implication: energy transfer can be increased when τ_{pst} is increased
- Theory: increasing energy transfer (τ) to *stable modes* $\Rightarrow$ lowers transport
- Nonlinear frequency shift small ($\Delta\omega \propto k^2$) $\Rightarrow \tau_{pst} \approx \mathrm{i}/\left(\omega_t + \omega_i - \omega_j^*\right)$

  $\Rightarrow$ Obtain ω_i from *linear* problem: $\mathcal{L}\beta_i = \omega_i\beta_i$
- Construct a "nonlinear" turbulence optimization metric from linear theory

Optimizing for reduced turbulence

- Key optimization quantities: τ_{pst}, C_{pst}
- Stable mode saturation metric algorithm: (`gitlab.com/bfaber/PTSM3D`)

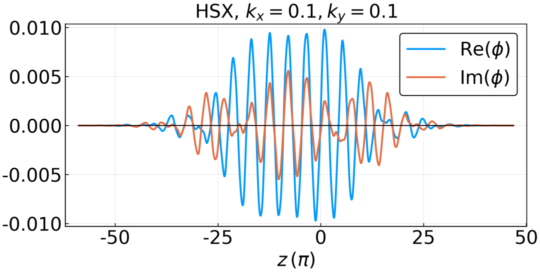
1. Solve linear eigenvalue problem on a (k_x, k_y) grid for $k_x, k_y \leq 1$
   - Three fields $\Rightarrow$ three roots: unstable, stable, marginally stable
2. Compute all combinations of τ_{pst}, C_{pst} involving at least one unstable, stable mode satisfying

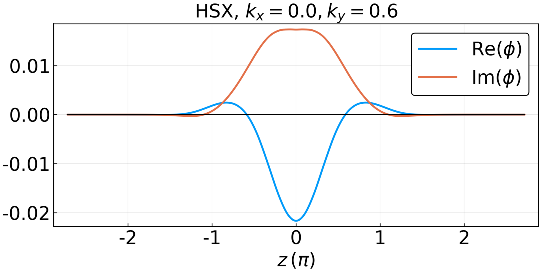
$$k_x - k_x' - k_x'' = 0, \quad k_y - k_y' - k_y'' = 0$$

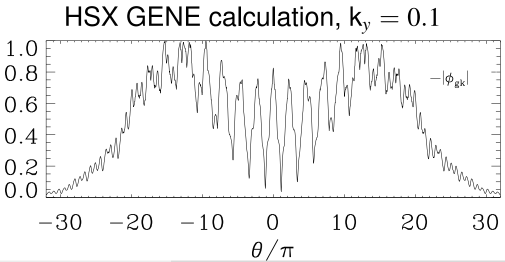
3. Break different contributions into zonal ($k_y' = 0$) and non-zonal ($k_y' \neq 0$)
4. At each (k_x, k_y), keep triplet that *maximizes* $|\tau_{pst} C_{pst}|$
5. Define a metric for optimization:

$$\langle|\tau_{pst} C_{pst}|\rangle_k = \sum_{k_x, k_y} \mathcal{S}_G(k_x, k_y) \left(|\tau_{pst}(k_x, k_y) C_{pst}(k_x, k_y)|^{\text{max}}_{\text{zonal}} + |\tau_{pst}(k_x, k_y) C_{pst}(k_x, k_y)|^{\text{max}}_{\text{non-zonal}} \right)$$

Model accurately captures geometry dependence

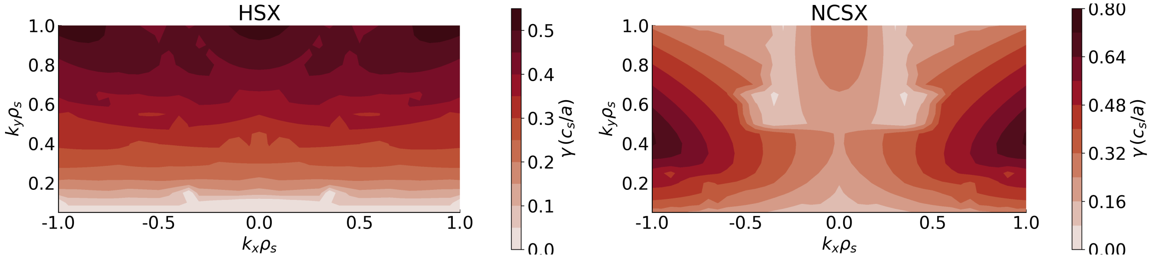






- Model accurately captures extended modes at low k_y; localized ballooning modes at higher k_y
- Essential to accurately computing ω_i, τ_{pst} and $C_{k,k'}$

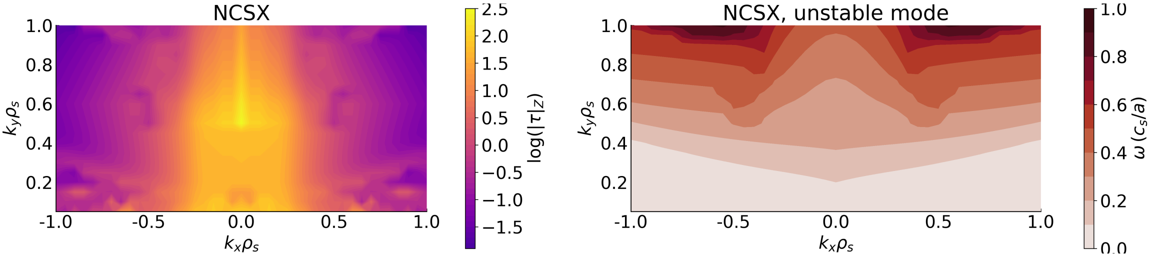
HSX, NCSX show substantially different saturation behavior



- Growth rate spectra substantially different between HSX, NCSX
  - Weakly k_x-dependent ITG in HSX (slab-like), strongly k_x-dependent ITG in NCSX (toroidal-like)
  - Magnetic shear at half-toroidal flux surface: HSX $\sim$ -0.05, NCSX $\sim$ -0.5
- Theory indicates different saturation mechanisms
- Agrees with $Q \propto 1/\langle \tau C \rangle$

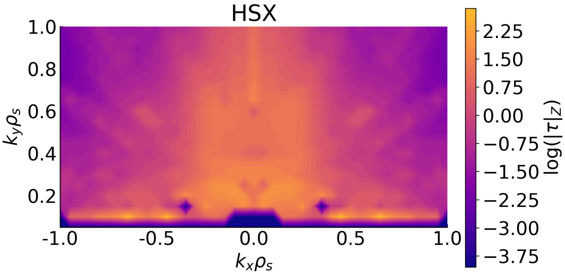
Geometry	$\langle \lvert \tau C \rvert \rangle_{\text{zonal}}$	$\langle \lvert \tau C \rvert \rangle_{\text{non-zonal}}$	Q_{GENE}
NCSX	10100	4725	15
HSX	7650	34018	5

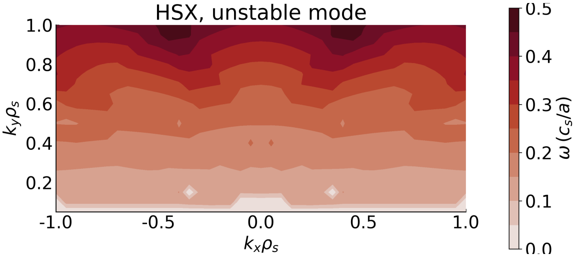
Zonal triplet lifetimes in NCSX



- Energy transfer through zonal modes matches linear instability region
- Zonal correlation times maximized when $\omega(k) - \omega^*(k') \approx 0$ $(\omega_Z \approx 0)$
- Falls off rapidly due to strong k_x dependence

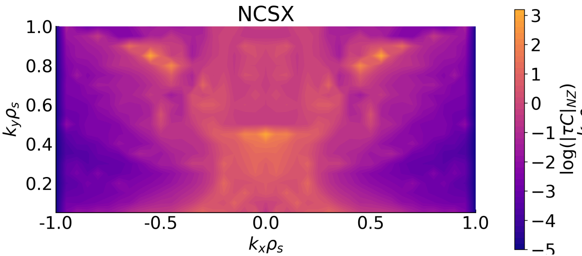
Zonal triplet lifetimes in HSX

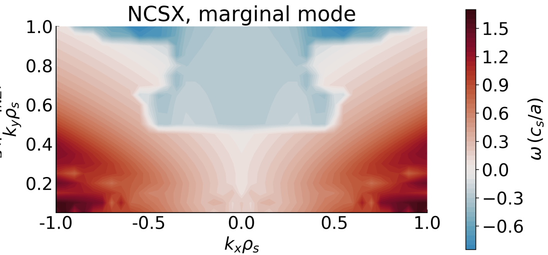




– Large zonal correlation times across k_x range at low k_y compared to NCSX
– Correlates with regions of weak k_x dependence in growth rates, frequencies

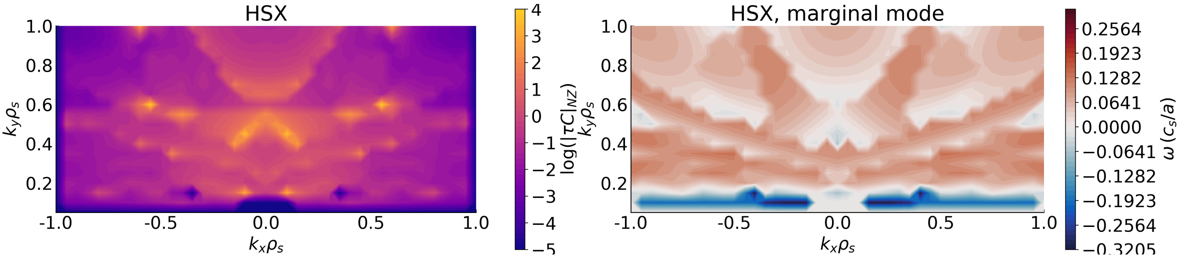
Non-zonal triplet lifetimes in NCSX





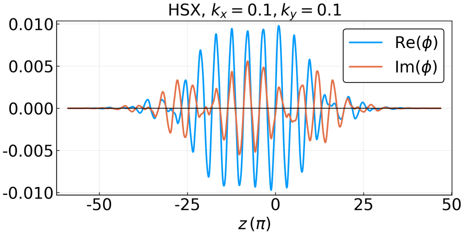
– Maximum non-zonal correlation times show strong localization in space
– Follows closely the contours where marginal mode frequency is zero (acts like zonal mode)
– Strong k_x-dependence in growth rates, marginal mode frequencies limits correlation times

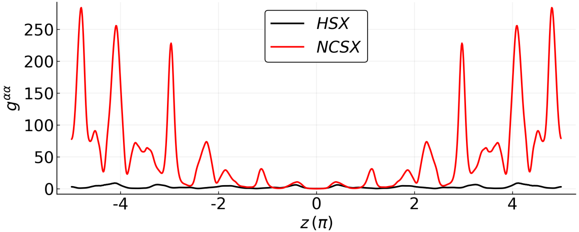
Non-zonal triplet lifetimes in HSX



- HSX marginal mode frequencies substantially closer to zero than in NCSX
  - Enables substantially larger correlation times compared to NCSX
- Largest $|\tau C|_{\text{non-zonal}}$ achieved when marginal mode frequencies are zero
- Weak dependence of ω on k_x enables large correlation times across the entire spectrum in HSX

Magnetic shear plays an important role





- HSX: small magnetic shear allows “slab-like” modes with small marginal mode frequencies, weak k_x dependence
  - Property general to small magnetic shear configurations (Bhattacharjee, *et al.*, PoF, 1983, Plunk, *et al.*, PoP, 2014)
- Zero-frequency marginal modes act similar to zonal modes, but at *non-zero* k_y
- Enables significantly larger space of triplet interactions, energy transfer to stable modes $\Rightarrow$ saturated turbulence smaller in HSX than NCSX despite larger linear growth rates

Summary

- A simple, three-field model of nonlinear ion turbulence has been implemented to explore turbulence saturation in stellartors through energy transfer to stable modes
- Energy transfer quantified by $\tau_{pst} = 1.0/\mathrm{i}\,(\omega_t + \omega_p - \omega_s^*)$
  - Break contributions into zonal, non-zonal coupled interactions
- Energy transfer, τ_{pst} strongly dependent on branch of ITG turbulence
  - Toroidal-like ITG dominated by zonal correlation times (NCSX)
  - Slab-like ITG dominated by non-zonal corelation times (HSX)
  - $\langle \tau C \rangle^{\text{HSX}}_{\text{non-zonal}} / \langle \tau C \rangle^{\text{NCSX}}_{\text{zonal}} \sim 3 \sim Q_{\text{NCSX}}/Q_{\text{HSX}}$
- Suggests slab-like modes and low magnetic shear are beneficial for nonlinear turbulence saturation

Future work

- Improve the physical fidelity of the underlying model, include effects of
  - KBM - see I. McKinney, P.86 on Thursday
  - TEM - see C.C Hegna, P.35 on Tuesday
- Include stable mode model in stellararator optimization framework
  - Mixing length proxy for turbulent transport with τC corrections:

$$Q \propto \frac{1}{\langle \tau C \rangle} \frac{\gamma}{k_\perp^2}$$

- Compare with nonlinear energy transfer results from GENE