Numerical Evaluation of 3D Local Equilibria

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Local 3D equilibrium allows investigation of shaping effects on local instability analysis.

- Affects of 3D geometry on local instabilities
    - Ballooning
    - Drift wave turbulence

- Global codes use lots of resources and time computing full plasma volume
- Generating MHD equilibrium for single flux surface would be much faster
    - 64x64 grid $\sim$ 4s, 128x128 $\sim$ 50s vs. hours for high resolution VMEC
    - Especially helpful for optimization

Local equilibrium theory generates MHD solutions on a flux surface.

- MHD equilibrium solutions generated satisfying
$$\mathbf{J} \times \mathbf{B} = \nabla p \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \nabla \cdot \mathbf{B} = 0$$
for a single flux surface $\psi = \psi_0$ in straight field line coordinates Θ and ζ

- Local equilibrium theory requires:

$\mathbf{X} = \mathbf{X}(\Theta, \zeta)$	Coordinate parameterization
$\iota\!\!\!-$	Rotational transform

and two of

$p' = dp/d\psi$	Pressure gradient
$\sigma = \mu_0 \langle \mathbf{J} \cdot \mathbf{B} \rangle / \langle B^2 \rangle$	Averaged parallel current
$\iota\!\!\!-' = d\iota\!\!\!-/d\psi$	Averaged magnetic shear

C. C. Hegna, *Phys. Plasmas*, 2000

NE3DLE is a tool to generate MHD equilibrium solutions.

- Written in Julia
  - Easy to use like Python or Matlab
  - Precompiles for faster execution
- Can handle analytic shaping and shaping files for R and Z
  - Helical axis w/ elongation
  - D-shaped axisymmetric (Miller equilibrium)
  - Read gridded shaping files or VMEC files
- May output surface or field line quantities for flux tubes

  https://gitlab.com/jduff2/NE3DLE.git

Key geometric quantities are associated with turbulence instability drive and damping.

- Integrated local shear Λ is stabilizing
  - $\Lambda = -\nabla(\Theta - \iota\zeta) \cdot \nabla\psi / B$
  - $g^{yy} = \frac{r^2 B^2}{|\nabla\psi|^2}(1 + \Lambda^2)$
  - Increase in g^{yy} enhances stabilization
- Curvature contributes to instability drive
  - Curvature $= rR_0 \frac{B}{|\nabla\psi|}(\kappa_n + \Lambda\kappa_g)$
  - Curvature < 0 enhances ITG drive
  - Curvature > 0 reduces ITG drive
  - $\kappa_n = \hat{\mathbf{n}} \cdot (\hat{\mathbf{b}} \cdot \nabla)\hat{\mathbf{b}}$
  - $\kappa_g = (\hat{\mathbf{b}} \times \hat{\mathbf{n}}) \cdot (\hat{\mathbf{b}} \cdot \nabla)\hat{\mathbf{b}}$
  - $\hat{\mathbf{b}} = \mathbf{B}/B$, $\hat{\mathbf{n}} = \nabla\psi/|\nabla\psi|$

D-shaped axisymmetric equilibria can be described by nine independent parameters in the Miller Equilibrium.

$$R = R_0 + r\cos(\theta + (\arcsin\delta)\sin\theta) \qquad Z = \kappa r \sin(\theta)$$

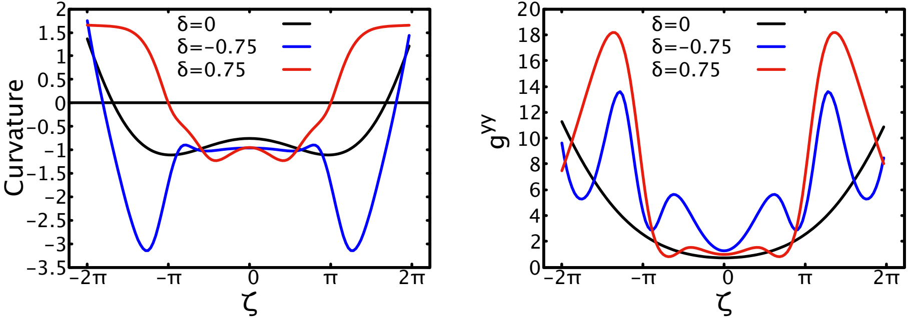
$$B_p = \frac{d_r\psi[\sin^2(\theta+x\sin\theta)(1+x\cos\theta)^2+\kappa^2\cos^2\theta]^{1/2}}{\kappa R[\cos(x\sin\theta)+d_r R_0\cos\theta+[s_\kappa-s_\delta\cos\theta+(1+s_\kappa)x\cos\theta]\sin\theta\sin(\theta+x\sin\theta)]}$$

$\sin x=\delta \qquad f(\psi)=RB_\phi$

κ	Elongation	$q=f/2\pi \int dl_p/R^2 B_p$	Safety Factor
δ	Triangularity	$\alpha=-(2\mu_0 r R_0 q^2/B_0)dp/d\psi$	Pressure Gradient
$A=R_0/r$	Aspect Ratio	$\hat{s}=d\ln q/d\ln r$	Magnetic Shear
$d_r R_0$	Shift	$s_\delta=\delta/(1-\delta)^{1/2}$	
		$s_\kappa=(\kappa-1)/\kappa$	

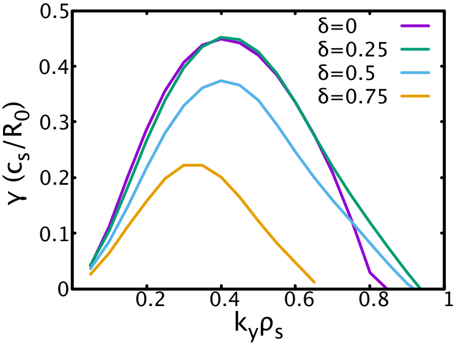
R. L. Miller, et al., *Phys. Plasmas*, 1998

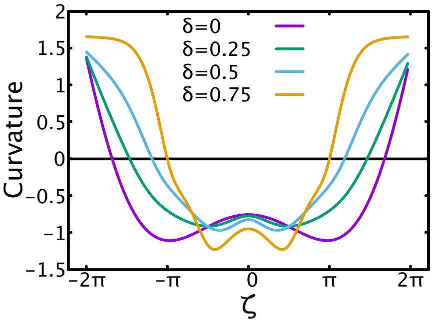
Varying δ modifies key geometric quantities.



- Positive δ decreased destabilizing curvature drive
- Negative δ increased destabilizing curvature drive, but increased local shear contributions stabilization

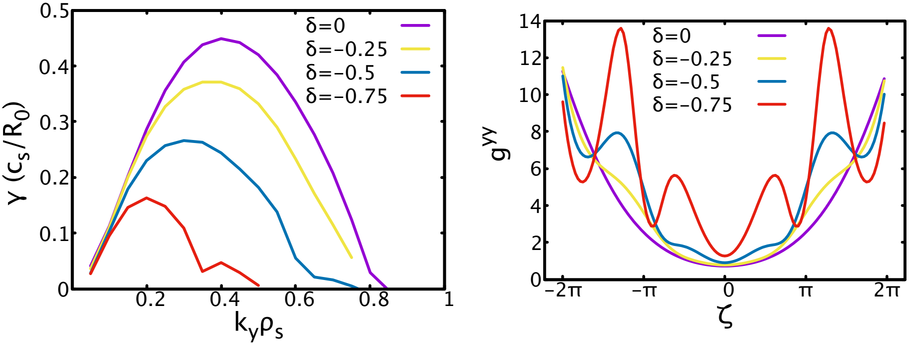
For positive δ, lower growth rates at low $k_y\rho_s$ with increasing δ.





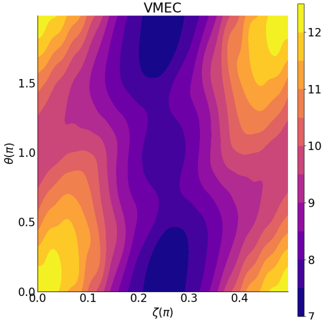
- Most turbulent transport occurs at low $k_y\rho_s$
- More favorable curvature with increased δ
- Lower growth rates at low $k_y\rho_s$ correlated with lower curvature drive as δ increased

For negative δ, growth rates decreased at at low $k_y\rho_s$ as $|\delta|$ increased.



- Curvature favorability did not correlate with growth rates at low $k_y\rho_s$
- Increased g^{yy} correlated with increasing $|\delta|$
- Lower growth rates at low $k_y\rho_s$ correlated with increased shear damping as $|\delta|$ increased

NE3DLE has reproduced large scale structure in the Jacobian for HSX.



- Jacobian calculation in straight-field-line coordinates is not optimal
  - Noise from second derivatives dominates calculation when too many modes are included
  - Cannot currently reproduce fine-scale structure of VMEC
- Implement VMEC strategy with "optimal" poloidal angle to reduce power in large (m,n) modes

Future Work

- Understand instability physics
  - Which geometry factors important for growth rates, saturation physics
  - Which components of Fourier spectrum influence geometry factors
- Wrapped in an optimizer to optimize for ITG turbulence