Numerical optimization of permanent magnets for stellarators with simple coils

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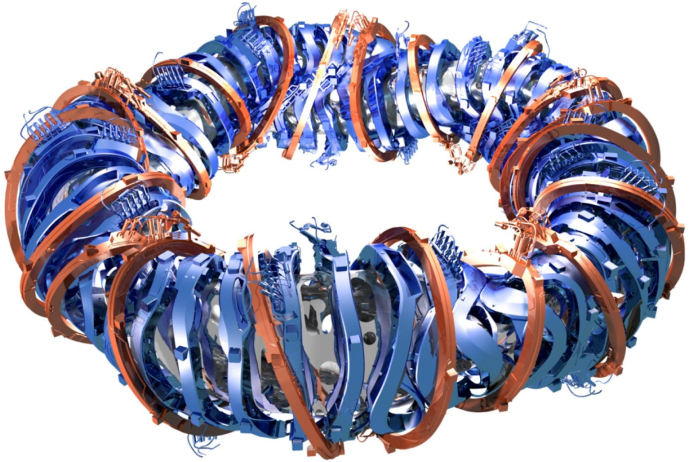
Stellarator coils are generally complicated.

- Due to the 3D nature of magnetic field, stellarators generally require non-planar coils.
- Difficulties in fabricating and assembling stellarator coils partly led to the cancellation of NCSX (Neilson IEEE 2010) and the delay of W7-X (Riße FED 2009).



Superconducting coil system of W7-X.

- Substantial effort has been devoted to simplifying stellarator coils.
  - **New coil design methods**: REGCOIL (Landreman NF 2017), FOCUS (Zhu NF 2018), winding surface opt. (Paul NF 2018), plasma-coil integrated opt. (Hudson PLA 2018).
  - **Better handling the tight tolerance**: Hessian matrix method (Zhu PPCF 2018, NF 2019), shape gradient (Landreman NF 2018), stochastic optimization (Lobsien NF 2018).

Permanent magnets provides an alternative way to generate magnetic field.

- Rare-earth magnets are boosted to have relatively high remanent field and good coercivity.
  - Nd-Fe-B magnet has a remanent field as high as Br=1.55T (Matsuura JMM 2006).
  - It can withstand reversed background magnetic field up to H_{ci}=5.0T when cooled down.

- Special arrangements can enhance magnet efficiency (Halbach NIM 1980). A 5.16 T magnetic field was reported (Kumada IEEE 2004) using neodymium magnets.

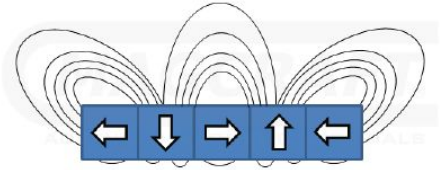
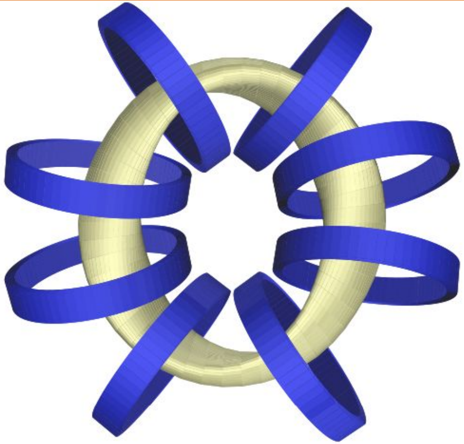


Illustration of 1D "one-sided flux" magnets.
Similar arrangements are used in fridge magnets.

- Commercially available in large amount with inexpensive price.

Helander's approach (Helander *et al.* 2020 Phys. Rev. Lett.)



Merkel's method for designing coils (NESCOIL): On a prescribed outer surface (winding surface) surrounding the plasma, a divergence-free surface current distribution is represented by current potential.

$$\mathbf{K} = \mathbf{n} \times \nabla \Phi$$

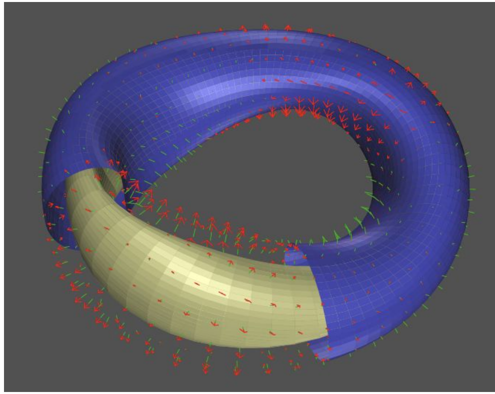
The surface current distribution is chosen by linearly minimizing the normal magnetic field on the plasma surface.

$$\chi_B^2 = \iint_{\partial P} (\mathbf{B} \cdot \mathbf{n})^2 \, \mathrm{d}S$$

$$\mathbf{B} = \mathbf{B}_{plasma} + \mathbf{B}_{fixed} + \mathbf{B}_K$$

$$\mathbf{B}_K = \frac{\mu_0}{4\pi} \iint_{\partial C} \frac{\mathbf{K} \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \, \mathrm{d}S'$$

$$\mathbf{B}_{\mathrm{m}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \left(\int_{\Omega} (\nabla \times \mathbf{M}) \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} dV' + \int_{\partial\Omega} (\mathbf{M} \times \mathbf{n}) \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} dS' \right)$$



If assuming $\nabla \times \mathbf{M} = 0$ in Ω and $\mathrm{M} = \begin{cases} -\nabla\Phi & \text{, on } \partial\Omega_{inner} \\ 0 & \text{, on } \partial\Omega_{outer} \end{cases}$, one can use NESCOIL solution directly.

Landreman's approach

Linear problem:

$$\min_{\mathbf{M}}\left(\int_{\text{plasma}} d^2x\left(\mathbf{B}\cdot\mathbf{n}\right)^2 + \lambda \int_{\text{PM region}} d^3x\left|\mathbf{M}\right|^2 wd \right) \qquad (1)$$

λ = regularization parameter

$w(\theta,\zeta)$ = weight (for ports)

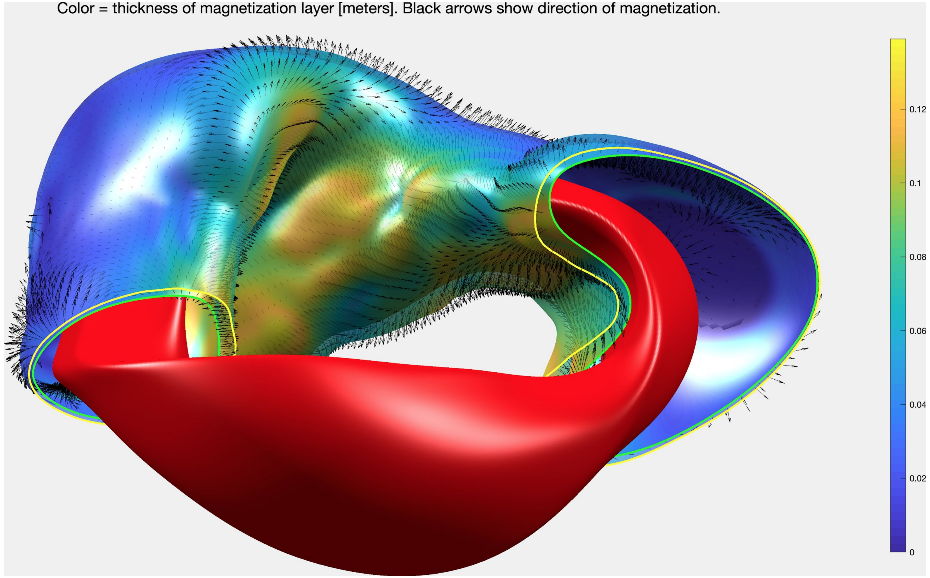
$d(\theta,\zeta)$ = thickness of PM layer

Iteration:

$$d_{j+1} = \frac{d_j}{M_{\text{target}}}\left|\mathbf{M}_j\right| \qquad (2)$$

$$\mathbf{B}_M(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3\mathbf{r}' \frac{1}{\left|\mathbf{r}-\mathbf{r}'\right|^3}\left(3\frac{\mathbf{M}(\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{\left|\mathbf{r}-\mathbf{r}'\right|^2} - \mathbf{M}(\mathbf{r}')\right)$$

$$\mathbf{M}(s,\theta,\zeta) = \sum_{j,k,\ell} M_{j,k,\ell}\, p_j(\theta,\zeta) I_k(s)\mathbf{e}_\ell(\zeta)$$



Solution of magnetization for vacuum NCSX boundary.

Linear method

Zhu *et al* 2019 arXiv: 1912.05144

Perpendicular dipole is equivalent to surface current potential.

(Recall Merkel's method) Surface current potential and its vector potential.

$$\mathbf{K} = \mathbf{n}' \times \nabla\Phi \qquad \mathbf{A}_K = \frac{\mu_0}{4\pi}\iint_{\partial C} \frac{\mathbf{K}}{r}\,\mathrm{d}S' = \frac{\mu_0}{4\pi}\iint_{\partial C} \frac{\Phi\mathbf{n}' \times \mathbf{r}}{r^3}\,\mathrm{d}S'$$

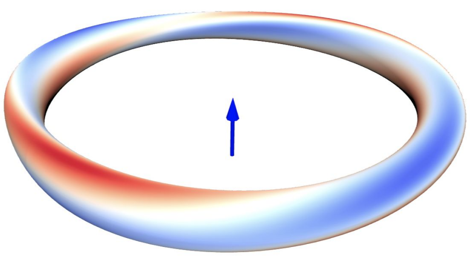
Discrete perpendicular dipoles can produce the same magnetic field (up to numerical precision).

$$\mathbf{M} = \Phi\mathbf{n}'\,\delta(s' - s_0') \qquad \mathbf{m}_{i_{\theta'},i_{\zeta'}} = \iiint \mathbf{M}\,\mathrm{d}V' = \Phi\mathbf{n}'\,\Delta S'_{i_{\theta'},i_{\zeta'}}$$

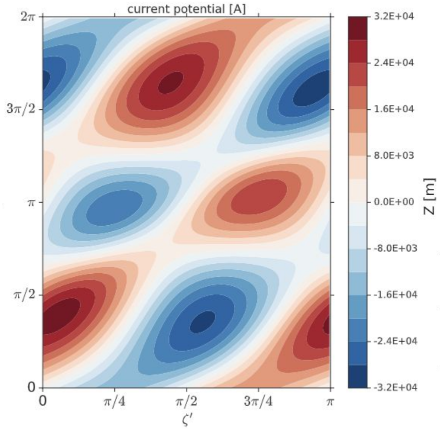
$$\mathbf{A}_{i_{\theta'},i_{\zeta'}} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}_{i_{\theta'},i_{\zeta'}} \times \mathbf{r}}{r^3} \qquad \mathbf{A}_M = \sum_{i_{\theta'}}\sum_{i_{\zeta'}} \mathbf{A}_{i_{\theta'},i_{\zeta'}} = \frac{\mu_0}{4\pi}\sum_{i_{\theta'}}\sum_{i_{\zeta'}} \frac{\Phi\mathbf{n}' \times \mathbf{r}}{r^3}\Delta S'_{i_{\theta'},i_{\zeta'}}$$

Numerical validation using a rotating elliptical stellarator.

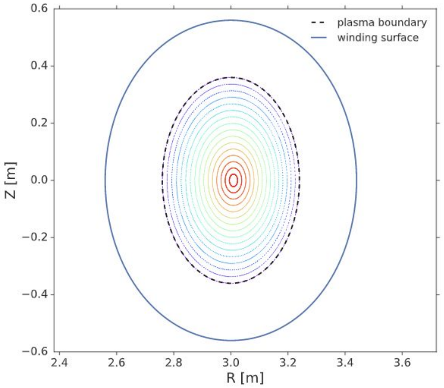
- Classical rotating elliptical stellarator, R=3.0m, r=0.3m, N_{fp}=2.
- Central current providing toroidal field, Ip=10 MA, B_0=0.61 T.
- If using one single layer surface magnetization, on a uniformly expanded surface (Δr=0.2m), the required maximum surface magnetization is about 3.2E4 A/m.



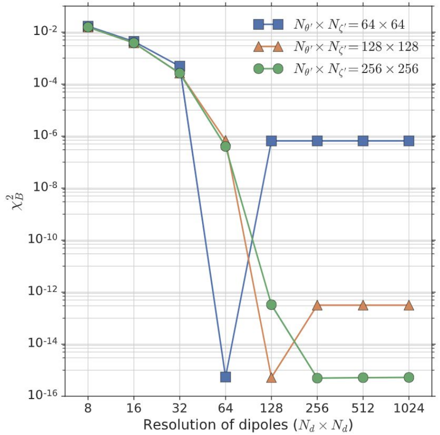
Plasma shape of the rotating ellipse. Toroidal field comes from a 10MA central current.



Current potential distribution on a uniformly expanded surface with separation of 0.2m.



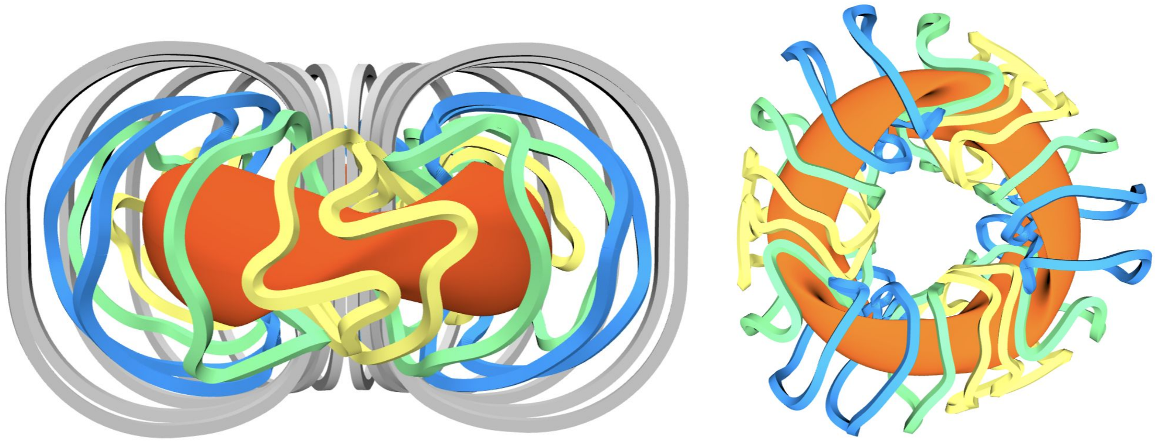
Poincare plots from field-line tracing in the magnetic field of toroidal field + 128x128 magnetic dipoles.



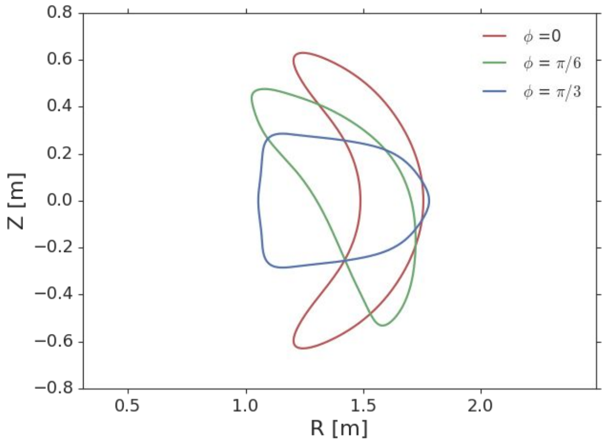
Convergence with different number of magnetic dipoles discretized from three current potentials

Half-Tesla NCSX equilibrium as the target.

- The actual NCSX consists of 18 modular coils (in 3 unique shapes), 18 TF coils and PF, CS coils.
- Only keep TF coils for providing the toroidal field (designed with 0.5T as the max. field).
- Scale NCSX C09R00 ($\langle B \rangle$=1.57T, β=4.1%) to $\langle B \rangle$=0.5T (same beta etc.). Required poloidal current I_{coils} is 3.77 MA. If using the actual TF coils, the current in each coil is about 0.21 MA.



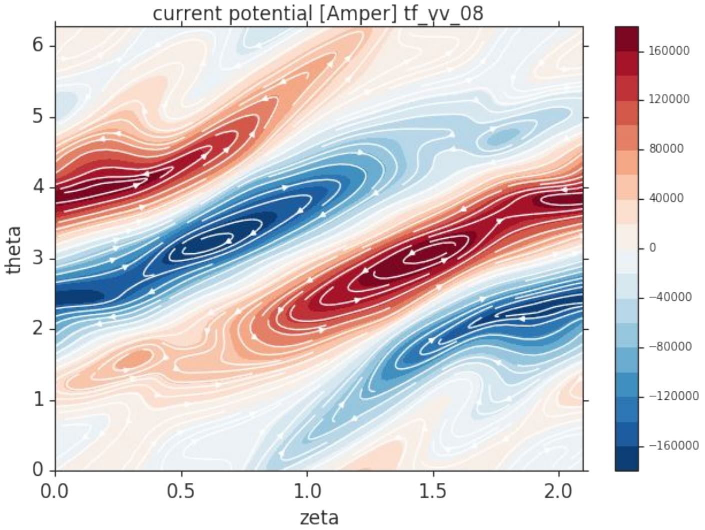
NCSX actual modular coils, TF coils and plasmas.



C09R00 configuration shapes at different toroidal angle.

Surface magnetization for NCSX.

1. Input files for REGCOIL include Bn from plasma currents and TF coils subtracted.
2. Calculate current potential distributions on vacuum vessel that cancels the input normal field.
3. Discretize surface magnetization from current potential with 120x360 (in three periods).



Current potential distribution on vacuum vessel.

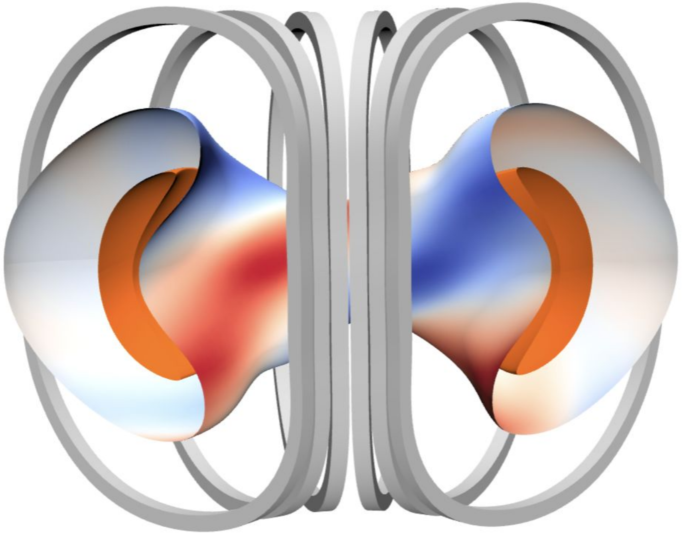
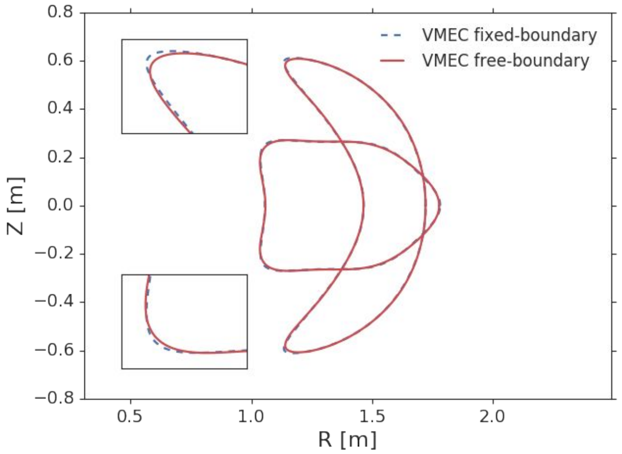


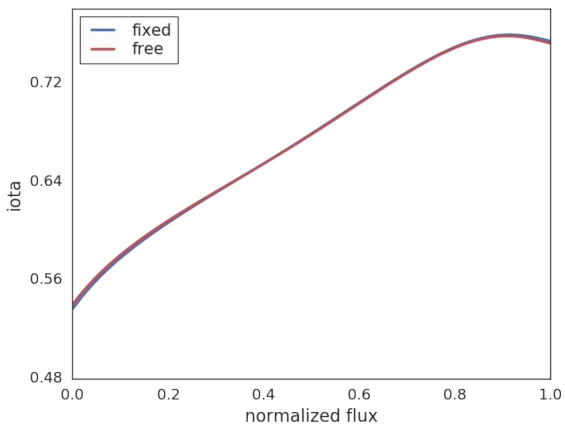
Diagram of TF coils plus surface magnetization on vacuum vessel.

Free-boundary calculations show highly accurate reconstructions.

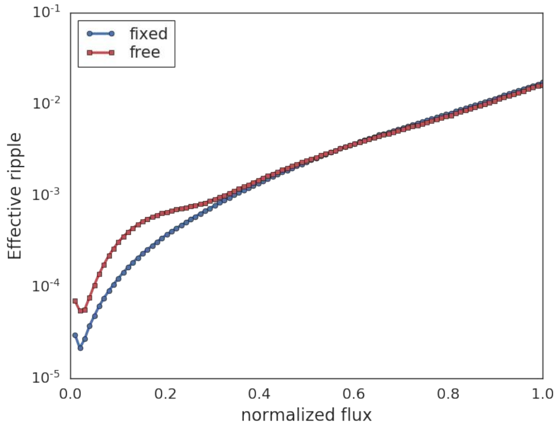
χ_B^2 is $1.49 \times 10^{-5} \mathrm{T}^2\mathrm{m}^2$



Comparisons of plasma surfaces.



Rotational transform profile comparisons.



Neoclassical transport comparisons.

Fixed: Half-tesla C09R00 in fixed-boundary mode
Free: Free-boundary VMEC with magnetic field from TF coils and surface magnetization on VV.

Finite thickness effect cannot be ignored.

- The only assumption made in previous calculations is to use the surface magnetization. This assumption is valid as long as the actual thickness of PM is much smaller than the distance between winding surface and plasma surface.
- The most popular commercially available PM is NdFeB, which has a residual magnetic field about 1.4T (magnetization limit ~ 1.1E6 A/m^2).
- For the half-Tesla NCSX configuration, to provide adequate magnetic field the maximum surface magnetization is about 1.8E5 A/m. Required thickness of NdFeB magnets to approximate such a surface magnetization is about 16 cm.
- This is somehow larger than the distance from plasma surface to vacuum vessel at some locations. In other words, the effect of finite-thickness cannot be negligible.

Iterative multi-layer implementation.

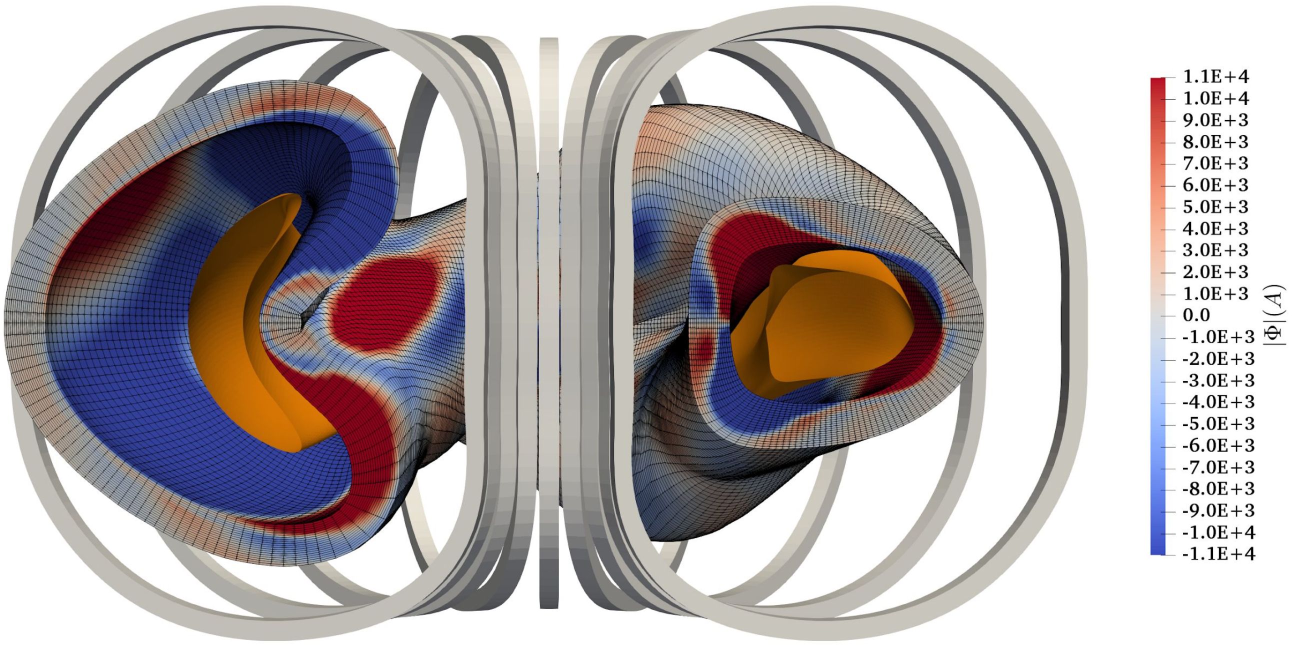
Instead of using one single winding surface, we choose several nested surfaces labeled as S'_1, S'_2, $\cdots$, S'_N from innermost to outermost and guarantee the gap h is sufficiently small to use the surface magnetization approximation.

Algorithm 1 Obtain permanent magnets with finite-thickness.

$i \leftarrow 1$
$\Phi_m \leftarrow B_r/\mu_0\, h$, where $h \ll d_{plasma-magnets}$
$\mathbf{B}_{fixed} \leftarrow \mathbf{B}_{coils}$
while $\chi^2_B >$ target_value **do**
  prepare winding surface S'_i
  $\min\limits_{\Phi_i} \iint_{\partial P} \left[(\mathbf{B}_{plasma} + \mathbf{B}_{fixed} + \mathbf{B}(\Phi_i)) \cdot \mathbf{n}\right]^2 \mathrm{d}S + \lambda\chi^2_R$, where χ^2_R is some regularization term and λ the weight
  if $\Phi_i(\theta', \zeta') < \Phi_m$ **then**
    $\mathbf{M}_i(\theta', \zeta') \leftarrow \Phi_i(\theta', \zeta')\mathbf{n}'_i$
  else
    $\mathbf{M}_i(\theta', \zeta') \leftarrow \Phi_m \mathbf{n}'_i$
  end if
  $\mathbf{B}_{fixed} \leftarrow \mathbf{B}_{fixed} + \mathbf{B}(\mathbf{M}_i)$
  $i \leftarrow i + 1$
end while

Algorithm for multi-layer implementation.

NCSX with only TF coils and PM with achievable magnetization.



The half-Tesla NCSX configuration with only TF coils and the permanent magnets. Only half of the torus are shown. The C09R00 plasma boundary are shown in orange. Colors indicate the magnitude of current potential Φ and negative values mean pointing outwards.

Field error is 9.02×10^{-5} T^2m^2. The total magnetic moment is equal to a NdFeB magnet of 3.6 m^3.

Nonlinear topology optimization

Problem description

Given a desired magnetic field, how can we come up with an "appropriate" design for permanent magnets that is attainable with present material?

$$\min \ \chi_B^2(\mathbf{M}) + \lambda f_R(\mathbf{M}), \quad \text{s.t. } |\mathbf{M}| \leq M_0$$

➔ The fitness of the magnetic field is normally evaluated by a boundary condition

$$\chi_B^2 = \iint_{\partial P} (\mathbf{B} \cdot \mathbf{n})^2 \, \mathrm{d}S \qquad \mathbf{B} = \mathbf{B}_{plasma} + \mathbf{B}_{fixed} + \mathbf{B}_M$$

➔ Appropriate designs should embrace the engineering constraints, like using the minimum amount of magnets, explicit "stay way" region, easy-to-build, etc.

Designing PM for stellarators is identified as a topology optimization problem.

Topology optimization is a mathematical method that optimizes material layout within a given design space, for a given set of loads, boundary conditions and constraints with the goal of maximizing the performance of the system.

The magnets have a explicit design space: outside the vacuum vessel (avoid melting), not too far away from the plasma (~10^{-5} order at r=0.5m).

- Flexible Advanced Magnets Used for Stellarators (FAMUS) is developed, based on FOCUS (*Zhu et al., NF 2018*; https://princetonuniversity.github.io/FOCUS/).
- Employing L-BFGS-B (Quasi-Newton) method for constrained optimization with analytically calculated gradient (Simulated Annealing is also available).
- Parameters are separable. Efficiently parallelized in permanent magnets using MPI.

FAMUS for designing permanent magnets

Use magnetic dipoles to calculate the magnetic field.

$$\mathbf{B}_M = \frac{\mu_0}{4\pi} \sum_{i=1}^{D} \left(\frac{3\mathbf{m}_i \cdot \mathbf{r}_i}{|\mathbf{r}_i|^5} \mathbf{r}_i - \frac{1}{|\mathbf{r}_i|^3} \mathbf{m}_i \right)$$

Split out the magnitude for better controlling the maximum allowable magnetization.

$$\rho = \begin{cases} 0 : \text{no material} \\ 1 : \text{material} \end{cases} \quad \rho = p^q \quad \mathbf{m}_i(p_i, \theta_i, \phi_i) = p_i{}^q \, m_{0i} \{ \sin\theta_i \cos\phi_i \mathbf{e}_x + \sin\theta_i \sin\phi_i \mathbf{e}_y + \cos\theta_i \mathbf{e}_z \}$$

Minimizing the normal field error with the constraint of least material using analytical gradients.

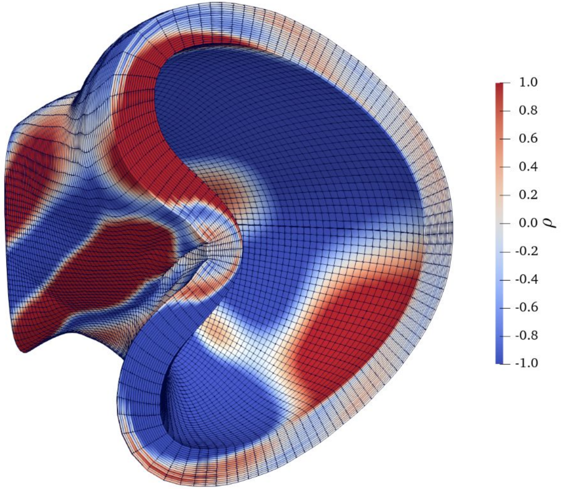
$$\min \quad F(p_i, \theta_i, \phi_i) = \iint_S \left[\mathbf{B}_M(p_i, \theta_i, \phi_i) \cdot \mathbf{n} - B_n^{tgt} \right]^2 \mathrm{d}a + \lambda \sum_{i=1}^{D} |\mathbf{m}_i(p_i, \theta_i, \phi_i)|^2$$

$$\text{s. t.} \quad p_i \in [0, 1], \; \theta_i \in [-\pi, \pi], \; \phi_i \in [-\pi, \pi], \; i = 1, \cdots, D \, .$$

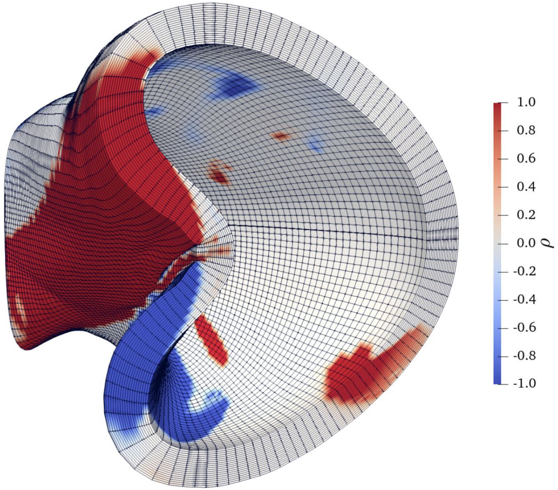
Each parameter is freely controlled to be varied or fixed, such that ports can be explicitly reserved and FAMUS can optimized the magnitude/density only or together with orientation.

Solution 1: Systematic optimization for perp. magnets.

- Allow only perpendicular orientations ; Reserve ports on the vacuum vessel.
- p∈[-1,1], q=7 to penalize intermediate values.
- 154843 free parameters, 200 iterations (takes 26 min. with 256 CPUs)



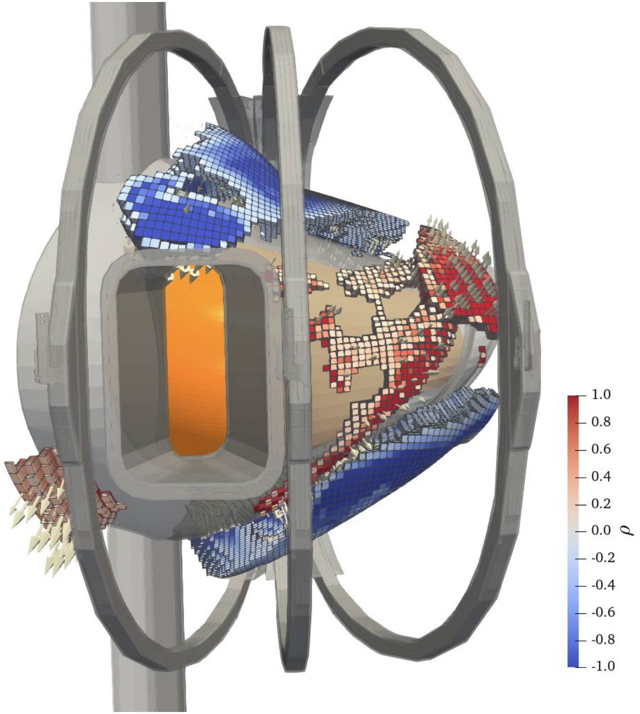
Finite-thickness solution with linear method (outside VV from 2cm ~ 22cm).



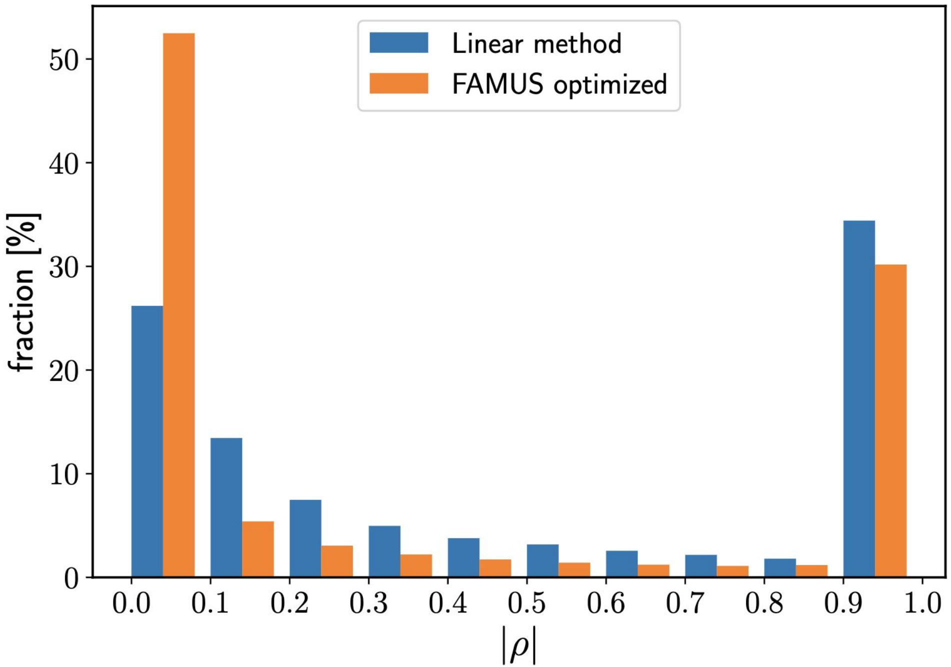
FAMUS optimized solution with the same position and orientation.

Comparison	Linear	FAMUS
f_B (chi^2_B)	3.99E-4	2.91E-5
Ave. |Bn/B|	1.10E-2	2.46E-3
Effective magnetization	7.53E5	5.49E5
Equivalent magnet volume	0.68 m^3	0.50 m^3

Massive outboard access is attractive.



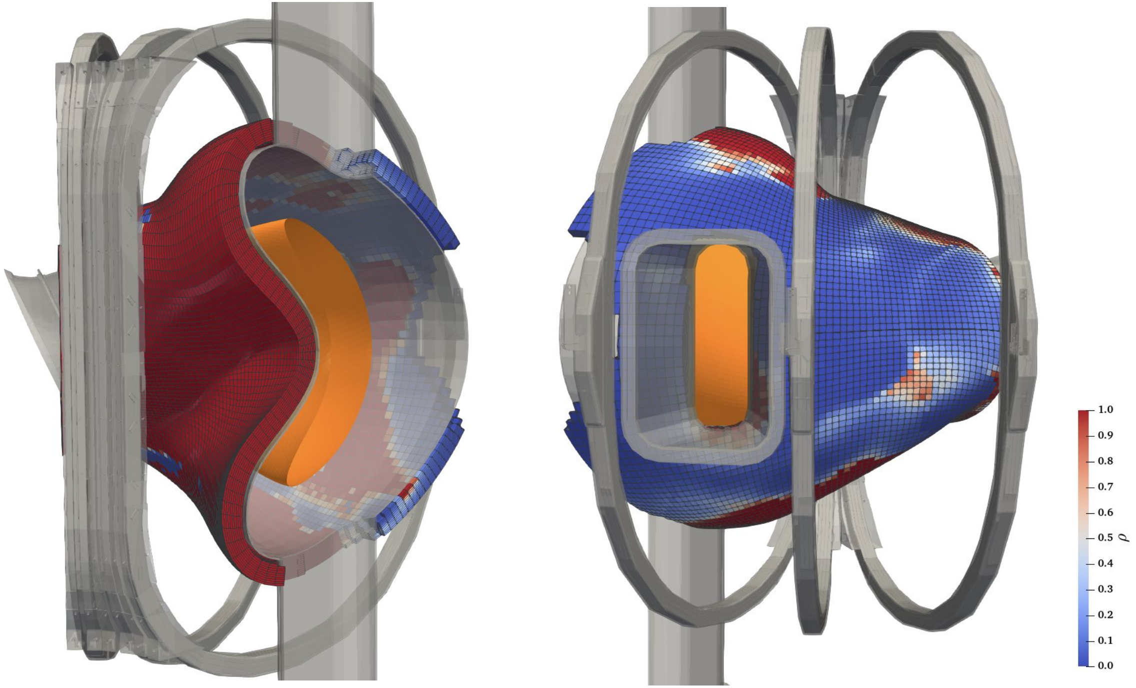
Clip the zeros (|ρ|<0.1), plotted with TF coils and vacuum vessel.



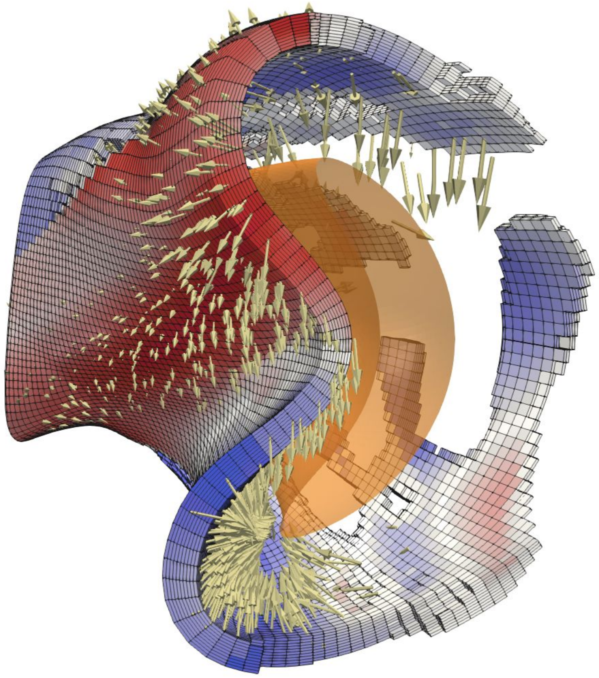
Comparison between the distribution of density for solutions from the linear method and FAMUS.

Solution 2: orientation optimization

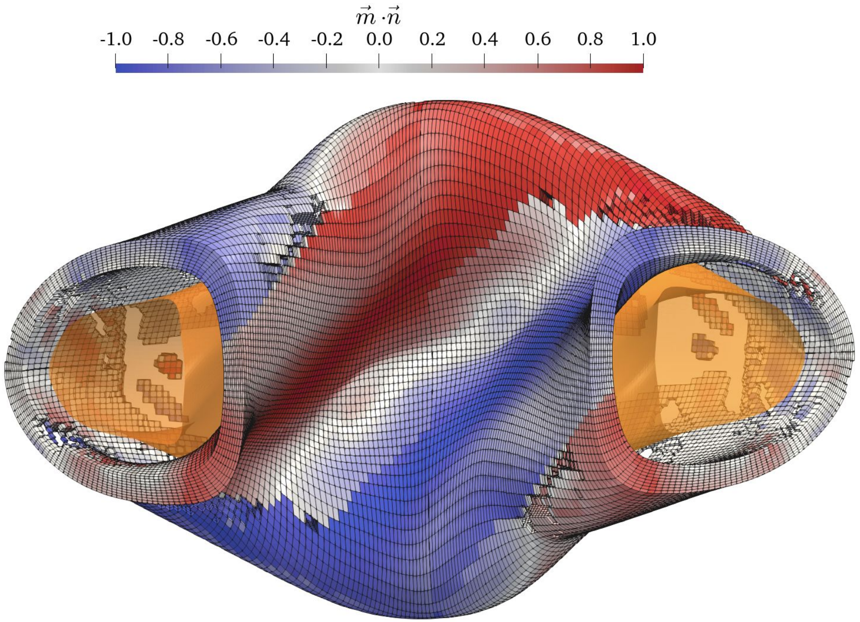
- Initial grid has the max. thickness ~ 10 cm.
- Allow the orientation to be varied.
- Reserve ports on the vacuum vessel.
- Minimize the normal field error and total amount of magnets.
- Equivalent volume ~ 0.42 m^3, field error ~ 6.6×10^{-6} T^2m^2.



Global halbach array is observed.

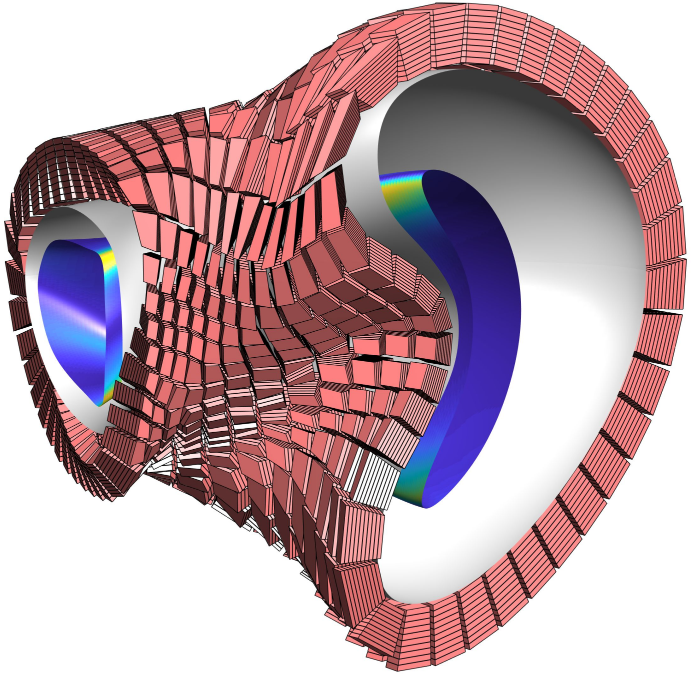


Orientations for each magnetic dipole.



Relative orientation compared to surface normal.

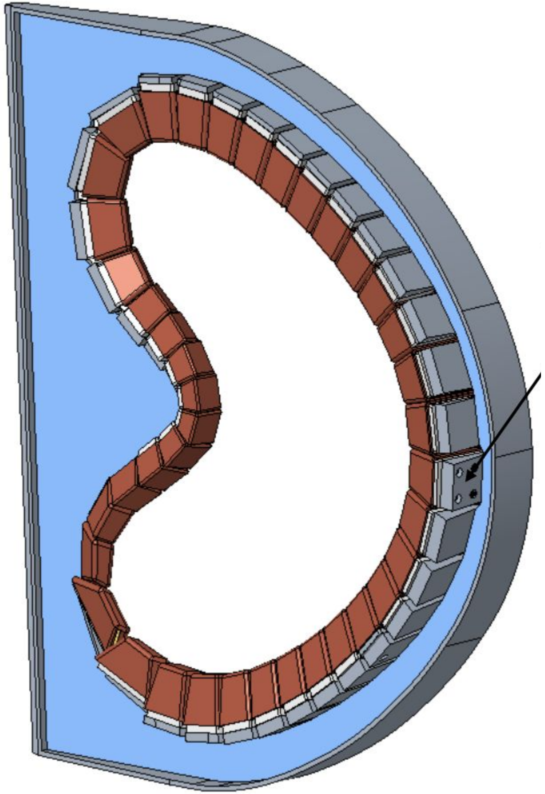
Solution 3: Trapezoidal arrangements for simple supporting.



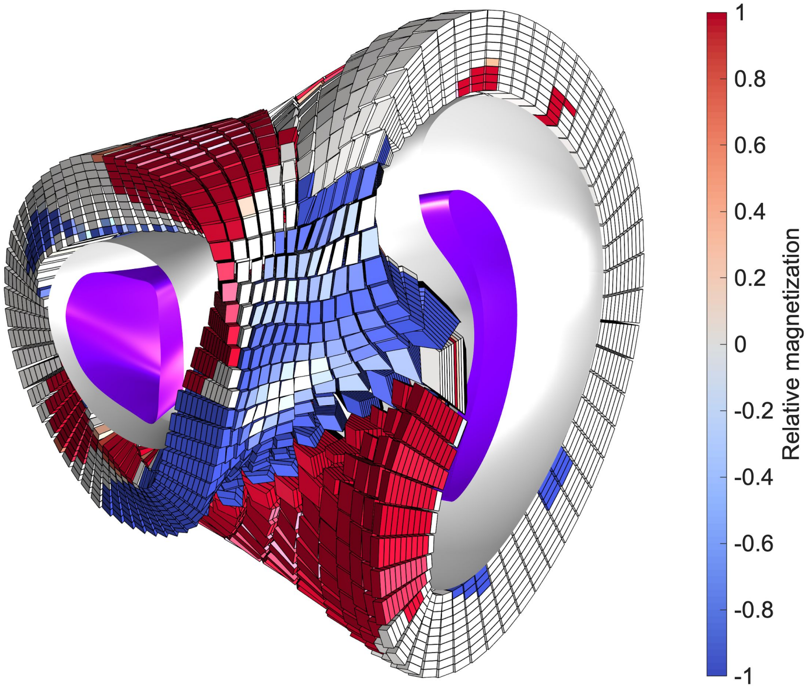
Magnets are aligned to be perpendicular to the vacuum vessel.

The orientation is restricted to be radial only (either inwards or outwards).

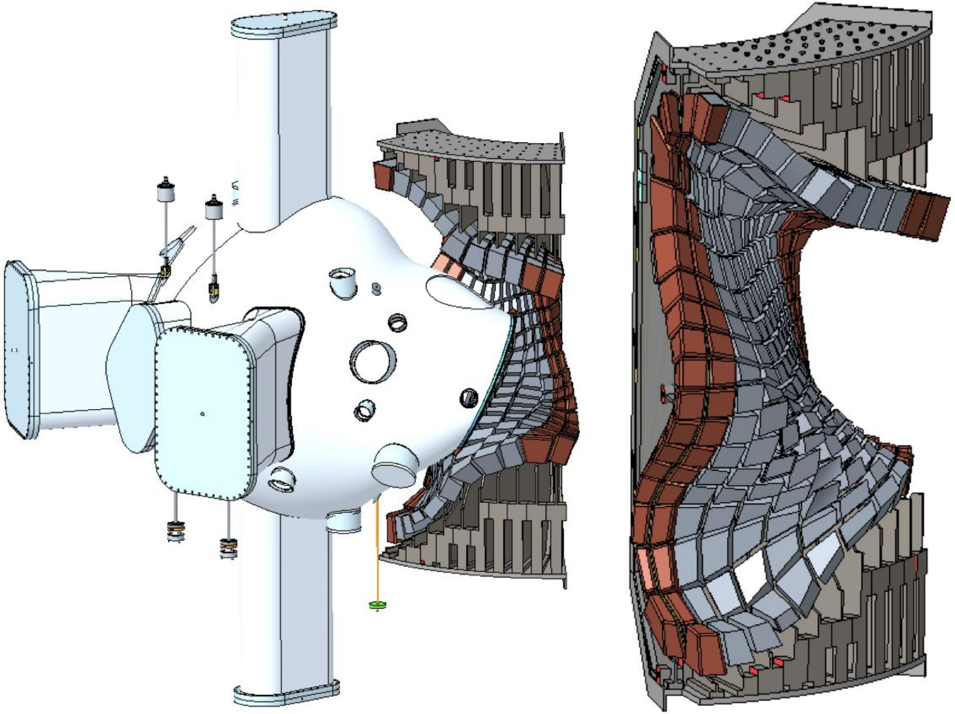
Mounting system is shown in the right.



Optimization successfully removes most outboard magnets.



Distribution of the density. Normal field error: 7.52E-4, effective moment: 7.28E5 Am^2 , equivalent volume: 0.66 m^3



Conceptual engineering designs for mounting

Summary

1. We have introduced two methods to optimize permanent magnets for stellarators, linear method and nonlinear topology optimization method.
2. Linear method is built on the base of current potential and is fast, robust, and can be used for quickly assessing the required amount of magnets, providing relatively good initial guess, and generating supporting field for other calculations.
3. Nonlinear topology optimization could find more engineering-practical solutions (max. allowable magnetization, reserving ports, Halbach arrays, min. magnet volume).
4. Half-Tesla shows that it is possible to produce optimized configurations using PM with extremely simple coils and it also provides massive access on the outboard side.

Discussions

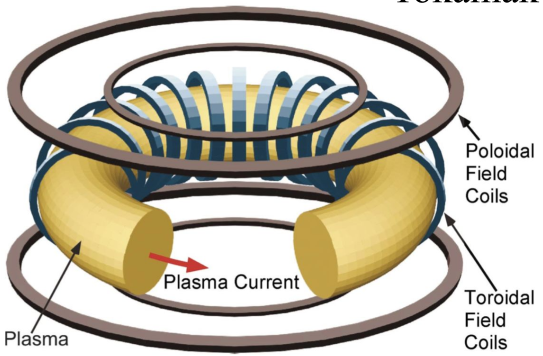
1. Balance between good field error and engineering simplicity is the main challenge.
2. Discretize optimization to allow 0^o, 90^o magnetization might be useful.
3. Applying global optimization methods on such a large-scale problem (DoF:1,000 ~100,000) to avoid local minima.
4. Better representation for the degrees of freedom to reduce dimensions?

Backup slides

What is a stellarator?

- A stellarator is a toroidal plasma confinement configuration that uses external coils to produce a non-axisymmetric magnetic field.
- Stellarator is an attractive approach to fusion energy.
  - Steady state operation
  - Low recirculating power
  - Free of disruptions
  - MHD stable
  - High density operation
- Stellarator used to have relatively bad neoclassical transport, which can be improved by optimizations.

Tokamak



Stellarator

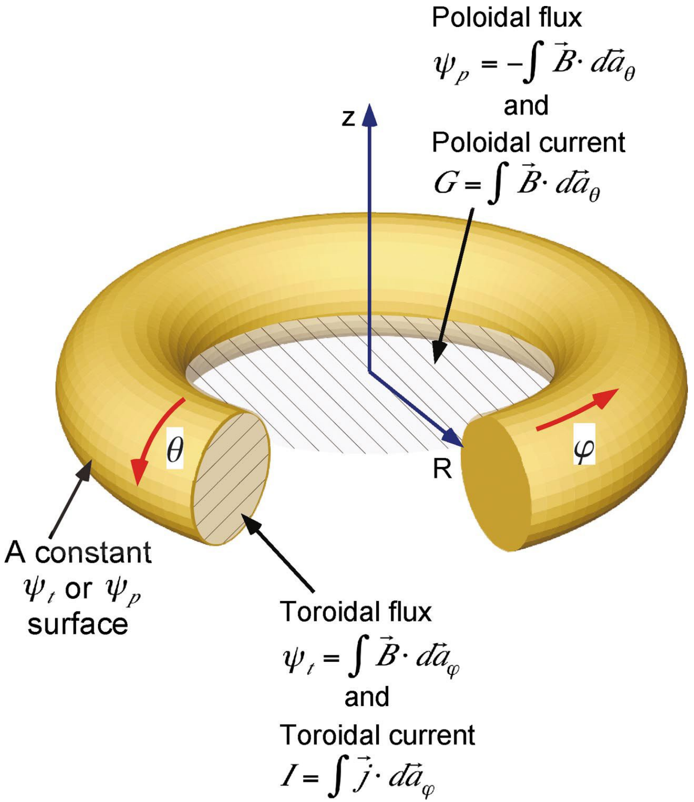
Boozer RMP 2004

Poloidal current is always required.

Ampere's Law:

- Choose the loop bounded by the minimum ψ_P surface;
- No magnetization in the plasma region;
- Line integral of B is normally non-zero → poloidal current is non-zero.

$$\oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_{free} + \oint_C \mu_0 \mathbf{M} \cdot \mathrm{d}\mathbf{l}$$

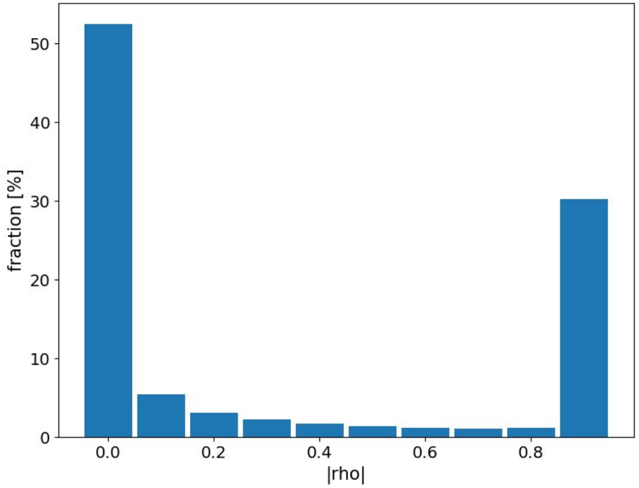


Poloidal and toroidal flux (*Boozer, RMP 2005*).

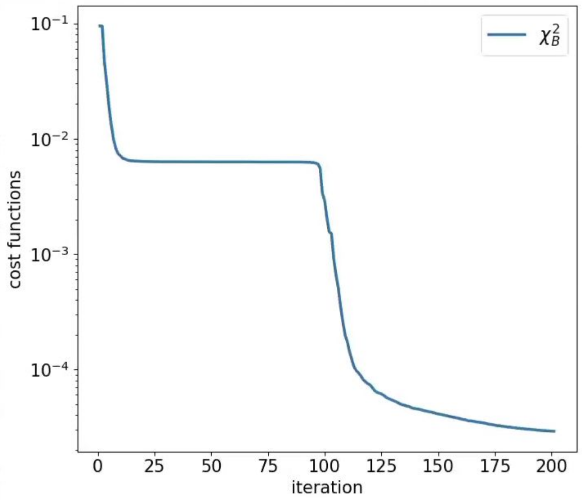
Details in deriving magnetic field from surface current.

$$
\begin{aligned}
\mathbf{A}_K &= \frac{\mu_0}{4\pi} \iint_{\partial C} \frac{\mathbf{n}' \times \boldsymbol{\nabla}\Phi}{r} \,\mathrm{d}S' \\
&= \frac{\mu_0}{4\pi} \iint_{\partial C} \left[\Phi \boldsymbol{\nabla} \times \left(\frac{\mathbf{n}'}{r} \right) - \boldsymbol{\nabla} \times \left(\frac{\Phi \mathbf{n}'}{r} \right) \right] \mathrm{d}S' \\
&= \frac{\mu_0}{4\pi} \iint_{\partial C} \left[\Phi \boldsymbol{\nabla} \frac{1}{r} \times \mathbf{n}' + \frac{\Phi}{r} \boldsymbol{\nabla} \times \mathbf{n}' - \boldsymbol{\nabla} \times \left(\frac{\Phi \mathbf{n}'}{r} \right) \right] \mathrm{d}S' \\
&= \frac{\mu_0}{4\pi} \iint_{\partial C} \left[-\Phi \frac{\mathbf{r}}{r^3} \times \mathbf{n}' - \boldsymbol{\nabla} \left(\frac{\Phi}{r} \right) \times \mathbf{n}' \right] \mathrm{d}S' \\
&= \frac{\mu_0}{4\pi} \iint_{\partial C} \frac{\Phi \mathbf{n}' \times \mathbf{r}}{r^3} \,\mathrm{d}S' + \frac{\mu_0}{4\pi} \iint_{\partial C} \mathbf{n}' \times \boldsymbol{\nabla} \left(\frac{\Phi}{r} \right) \mathrm{d}S' \\
&= \frac{\mu_0}{4\pi} \iint_{\partial C} \frac{\Phi \mathbf{n}' \times \mathbf{r}}{r^3} \,\mathrm{d}S' + \frac{\mu_0}{4\pi} \iiint_{C} \left[\boldsymbol{\nabla} \times \boldsymbol{\nabla} \left(\frac{\Phi}{r} \right) \right] \mathrm{d}V' \\
&= \frac{\mu_0}{4\pi} \iint_{\partial C} \frac{\Phi \mathbf{n}' \times \mathbf{r}}{r^3} \,\mathrm{d}S' \ .
\end{aligned}
$$

Perp. only design for the half-Tesla NCSX.



Intermediate values are penalized.



Convergence curve for the normal field error.