QH configuration WISTA_15
Beta dependency of the bootstrap current, ideal MHD ballooning stability limits

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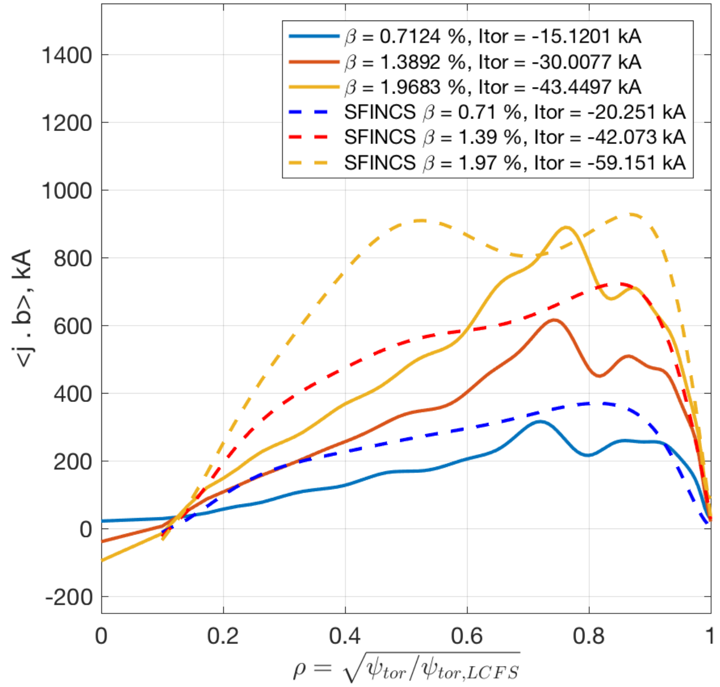
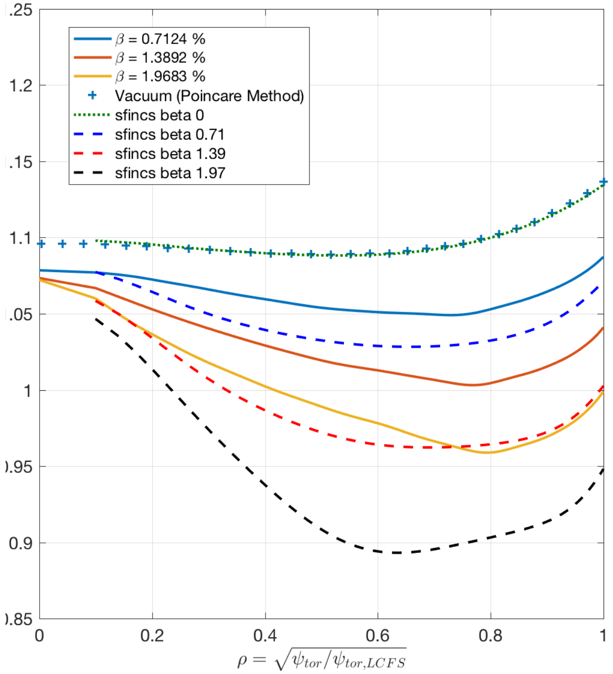
Finite beta, with self-consistent bootstrap current

- Part 1
  - Previous calculations show that BOOTSJ[1] underestimates the total bootstrap current by about 33%
  - The 'Beta scan' was repeated with SFINCS[2] for the bootstrap current calculation
    - Comparison to BOOTSJ
    - Beta scaling of neoclassical current, transform, epsilon effective
    - Equilibrium Limits, Mercier, Well, Ballooning
- Part 2
  - STELLOPT Stability optimization
  - Net toroidal current = 0 (no bootstrap current)
    - Stabilization to 1.8%
    - Increase in shear is associated with the stabilization of ideal ballooning modes

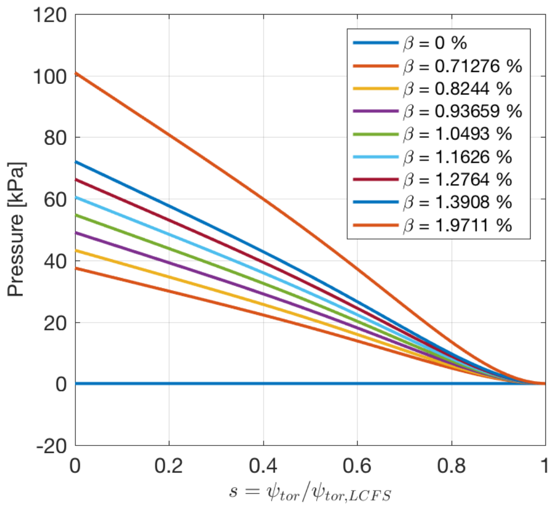
- Previously, it was shown that the self-consistent bootstrap current is underestimated by about 33% with BOOTSJ, compared to SFINCS

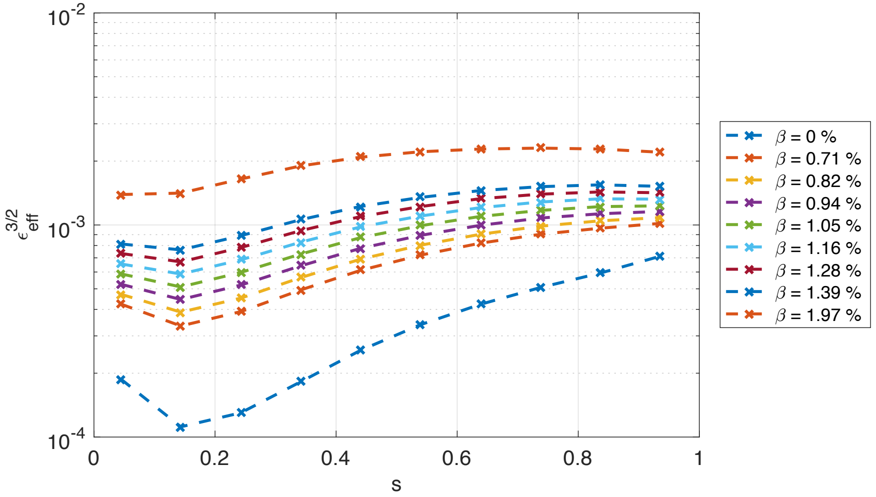
- A re-scan of Beta, using fixed temperature and density profile shapes, was performed with SFINCS
  - $T \sim T_0 * (1 - s)$
  - $N \sim N_0 * (1 - s^5)$
  - $P \sim P_0 (1 - s - s^5 + s^6)$

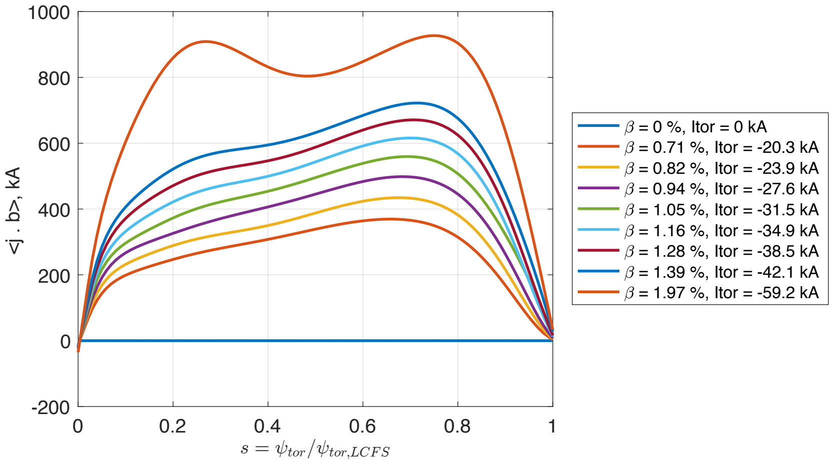


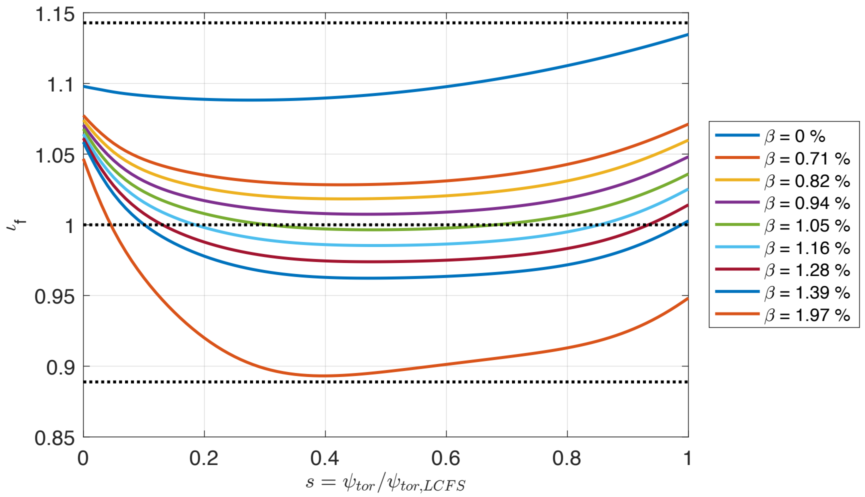


Results of the Beta scan with self-consistent bootstrap current (SFINCS) show that ATEN goes through iota=1 above 1%









Stability analysis (Mercier and Ideal MHD Ballooning)

Mercier Criterion

Mercier stability is given by[3,4]: $D_M = D_S + D_W + D_I + D_G \geq 0$

- Stabalizing shear term: $D_S \dfrac{\iota^2\pi^2}{s} = \dfrac{(\Psi''\Phi')^2}{4}$
- Well (or hill) term: $D_W \dfrac{\iota^2\pi^2}{s} = \int\int g d\theta d\zeta \dfrac{B^2}{g^{ss}}\dfrac{dp}{ds} \times \left(V'' - \dfrac{dp}{ds}\int\int g \dfrac{d\theta d\zeta}{B^2}\right)$
- Net current: $D_I \dfrac{\iota^2\pi^2}{s} = \left[\int\int g \dfrac{d\theta d\zeta}{g^{ss}} \dfrac{B^2}{g^{ss}} \Psi'' I' - (\Psi''\Phi') \int\int g \dfrac{(\boldsymbol{J}\cdot\boldsymbol{B}) d\theta d\zeta}{g^{ss}}\right]$
- Geodesic curvature: $D_G \dfrac{\iota^2\pi^2}{s} = \left[\int\int g \dfrac{(\boldsymbol{J}\cdot\boldsymbol{B}) d\theta d\zeta}{g^{ss}}\right]^2 - \left[\int\int g \dfrac{(\boldsymbol{J}\cdot\boldsymbol{B})^2 d\theta d\zeta}{g^{ss}B^2}\right] \int\int g \dfrac{B^2 d\theta d\zeta}{g^{ss}}$

Ballooning Stability

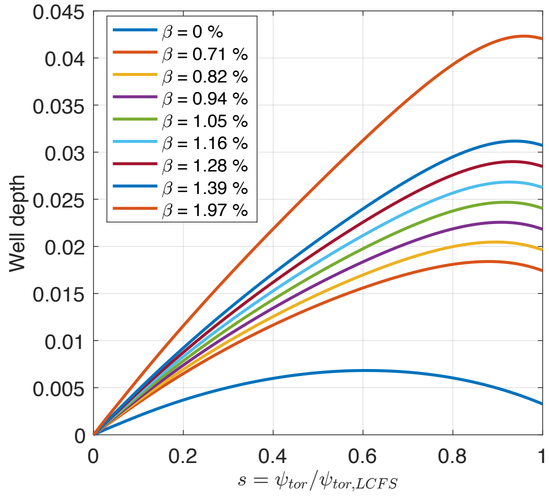
- Ideal ballooning mode growth rates are rapidly and accurately calculated using VMEC coordinates[5]

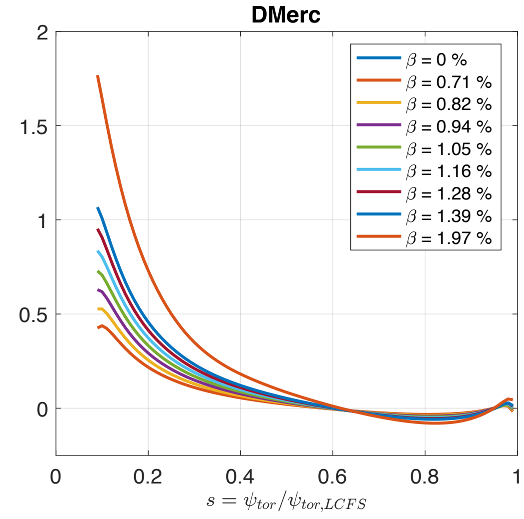
$$\alpha(s,\theta_v,\zeta_v) = \theta_v + \lambda(s,\theta_v,\zeta_v) - \iota\zeta_v$$

$$(\vec{B}\cdot\nabla)\left[\frac{|\nabla\alpha|^2}{B^2}(\vec{B}\cdot\nabla)\right]F + \left(\frac{R_0}{a}\right)^2 \frac{\beta_0 p'}{\Psi'^2}\kappa_s F = \gamma^2 \frac{|\nabla\alpha|^2}{B^2} F$$

Well depth and Mercier stability improves with Beta

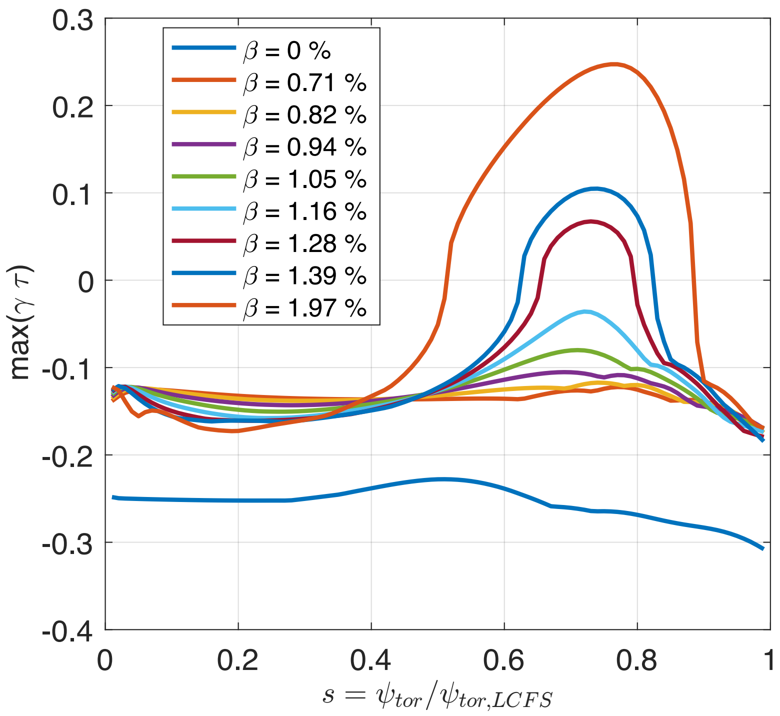
- The depth of the magnetic well increases with Beta
- A small 'magnetic hill' region exists near the LCFS
- Mercier stability improves with beta, except near the edge

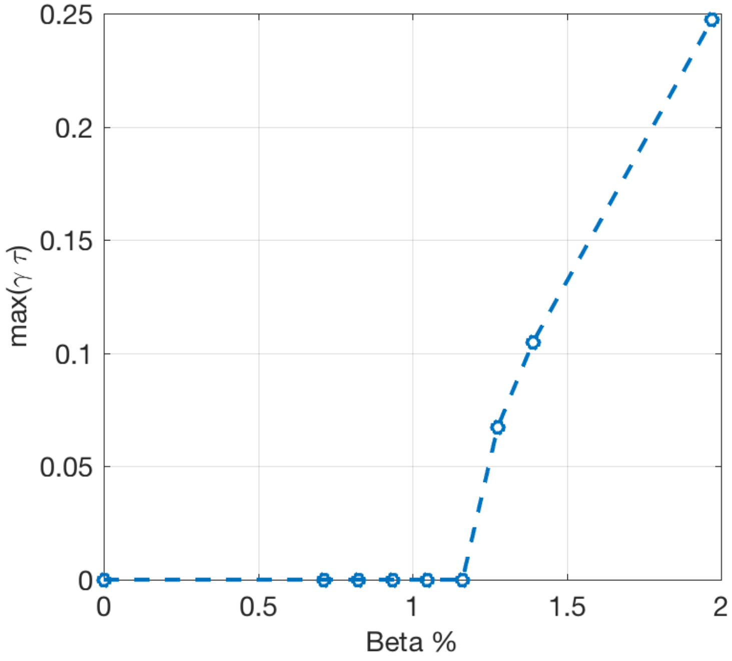




Ideal ballooning instability sets in around Beta=1.28%

- Ballooning growth rates are positive (unstable) near s=0.7 at Beta = 1.28%



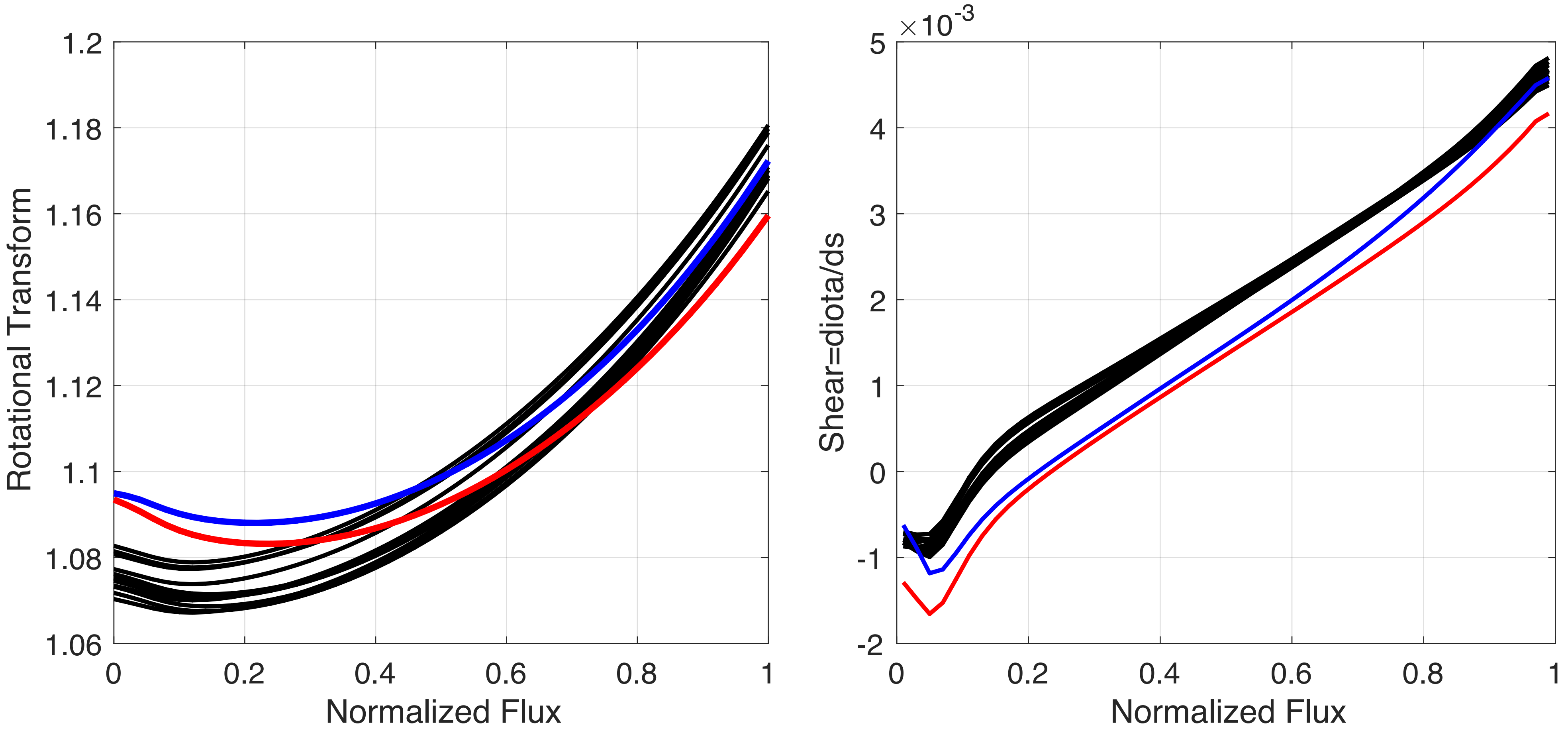


A ‘simple’ STELLOPT exercise

- STELLOPT Stability optimization
  - Initial condition: ATEN (ideal boundary, not with the WISTA_15 coil set)
  - Variables: RBS, ZBS (all modes up to ~8)
  - Fixed boundary equilibrium, Net toroidal current = 0 (no bootstrap current)
  - Targets: Aspect ratio, Major radius, Ballooning stability
  - Optimizer control parameters were modified between runs: EPSTOL and FACTOR
    - These two parameters control the finite differences and search vectors
    - Otherwise, the STELLOPT runs are identical
  - Results
    - Stabilization of ballooning modes through increased shear of the rotational transform.
    - Stabilization to 1.8%
    - Increase in shear is associated with the stabilization of ideal ballooning modes
    - Global shear is the quantity reported here
    - Local shear is very likely the more ‘precise’ hidden variable here,
      - Not directly accounted for in the optimization, except as part of the ballooning instability metric

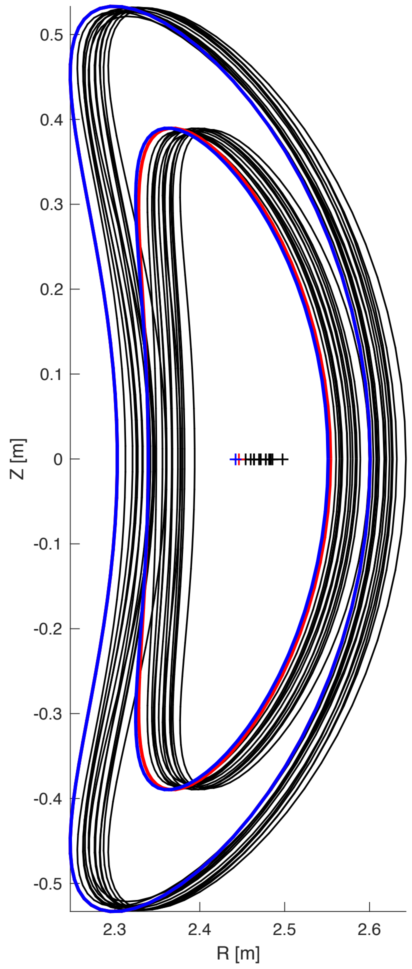
Blue: ATEN Vacuum
Red: Aten, Beta = 1.8%
Black: STELLOPT RUNS (about 10 ‘solutions’ shown)

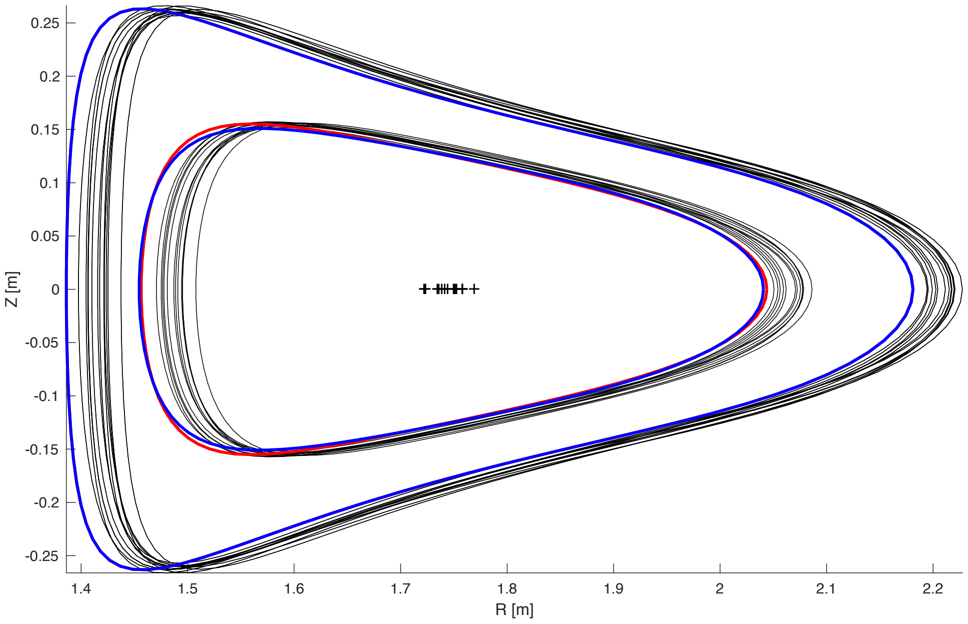
- Each of the STELLOPT runs is stable to Ideal MHD Ballooning modes
- The transform profile is unique for each run, but the shear is very similar



Blue: ATEN Vacuum
Red: Aten, Beta = 1.8%
Black: STELLOPT RUNS (about 10 ‘solutions’ shown)

- Each of the STELLOPT runs increases the major radius by a small amount





References

1. K.C. Shaing, et al., Phys. Fluids **B1** (1989), 148.
2. Landreman, Smith, Mollen, and Helander, Physics of Plasmas **21** (2014), 042503.
3. Bauer, F., Betancourt, O., Garabedian, P.A., Magnetohydrodynamic Equilibrium and Stability of Stellarators, Springer-Verlag, New York (1984).
4. B.A. Carreras et al, Nucl. Fusion **28** (1988), 1195.
5. R. Sanchez et al. /Computer Physics Communications **135** (2001), 82–92.