Ensuring equilibrium evaluations do not depend on parametrization

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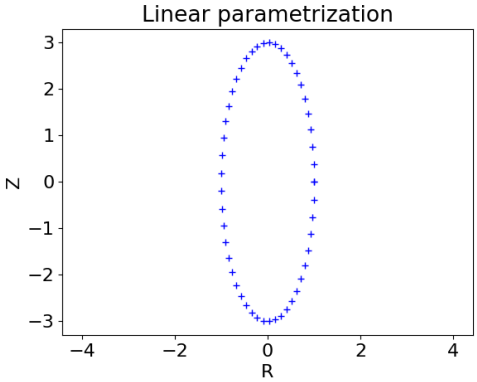
Parameter-dependent evaluations can cause problems

- STELLOPT and ROSE both optimize a boundary given by $R_{m,n}$ and $Z_{m,n}$ coefficients
- In this parametrization, the toroidal variable is the toroidal angle
- The poloidal variable is free, there is degeneracy in the representation
- VMEC internal surfaces resolve the degeneracy through a form of spectral condensation

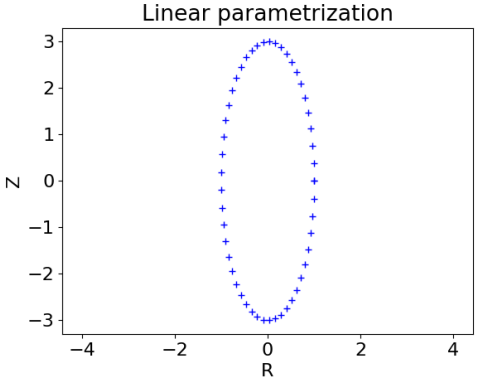
”Simple” linear parametrizations can skew results

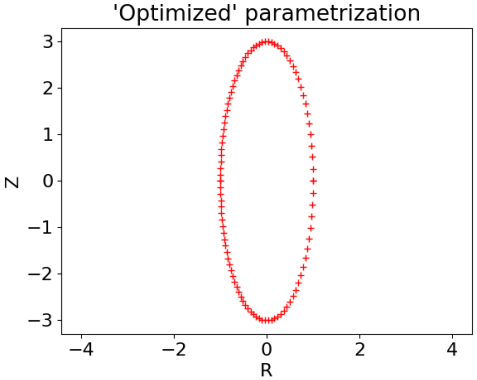


$$R = A\cos(t)$$
$$Z = B\sin(t)$$

- Let t be linear between [0,2π]
- Simple ellipse will have points overly represented near the elongated regions

Possible to ”optimize” a configuration without changing the boundary

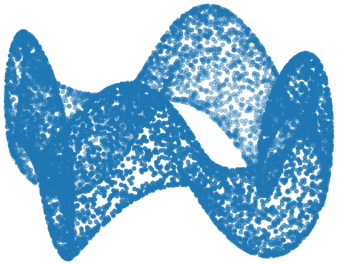




- Let Q be a property with lower penalties on the high-field side (neg. R)
- Exploiting the parametrization freedom can place more evaluation points in the ”good region”

Avoiding parametrization errors in energetic particle evaluation

- Early EP confinement calculations showed ”spurious” improvements
- Fixed by properly distributing particles in both physical and velocity space
- Generate randomized spawn points, such that the probability of finding particle in volume element $dV_0 \propto \mathcal{J}(s_0, \theta_0, \zeta_0)$



Early turbulent transport calculations showed similar problems

- Optimization using early turbulence metrics improved confinement by focusing on resonances
- Resonances were very deep, but also very steep
- Evaluation with slightly different settings showed that the improvements vanished

Possible issue with coil evaluation

- Typically coil-fits are evaluated by the normal field $\int_S B_n^2$
- Numerically, the integral is evaluated over a discrete sum
- Coils could be pushed to better fit oversampled regions (like the tops of the ellipses)
- Given linear parametrization in toroidal angle, QH configs, which have large radial excursions, will be overfit where R_{ax} is small (mid periods)

Ensuring Coil Evaluations do not Depend on Parametrization

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FOCUS Objective Functions Should be Parameterization Independent

- Fourier series coil parameterization in FOCUS is not an equal arc length parameterization and FOCUS does not use spectral condensation

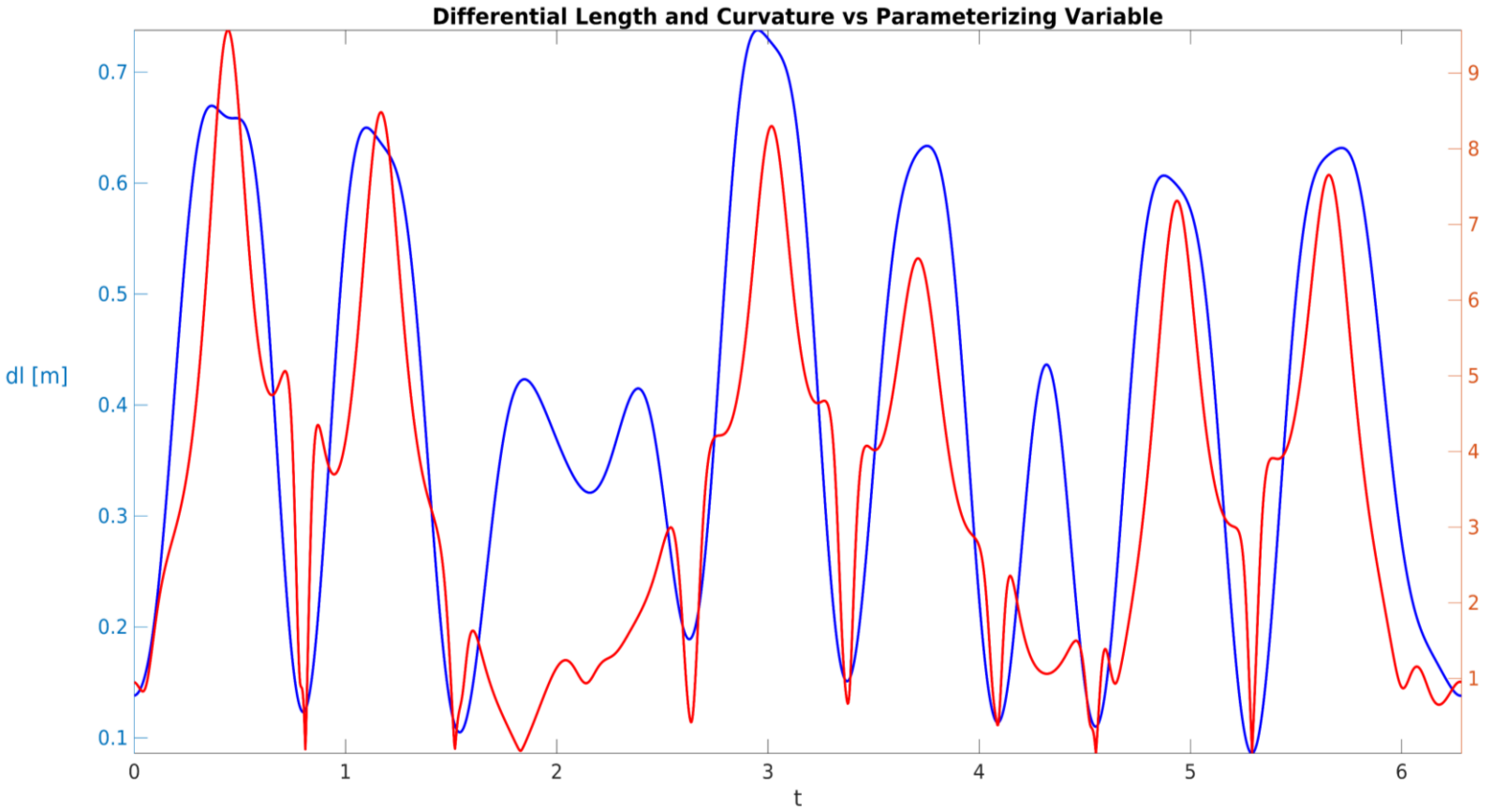
$$\boldsymbol{r} = \sum_{n=0}^{N_F} \left[\left(X_{c,n}\cos(nt) + X_{s,n}\sin(nt)\right)\hat{\mathbf{x}} + \left(Y_{c,n}\cos(nt) + Y_{s,n}\sin(nt)\right)\hat{\mathbf{y}} + \left(Z_{c,n}\cos(nt) + Z_{s,n}\sin(nt)\right)\hat{\mathbf{z}} \right]$$

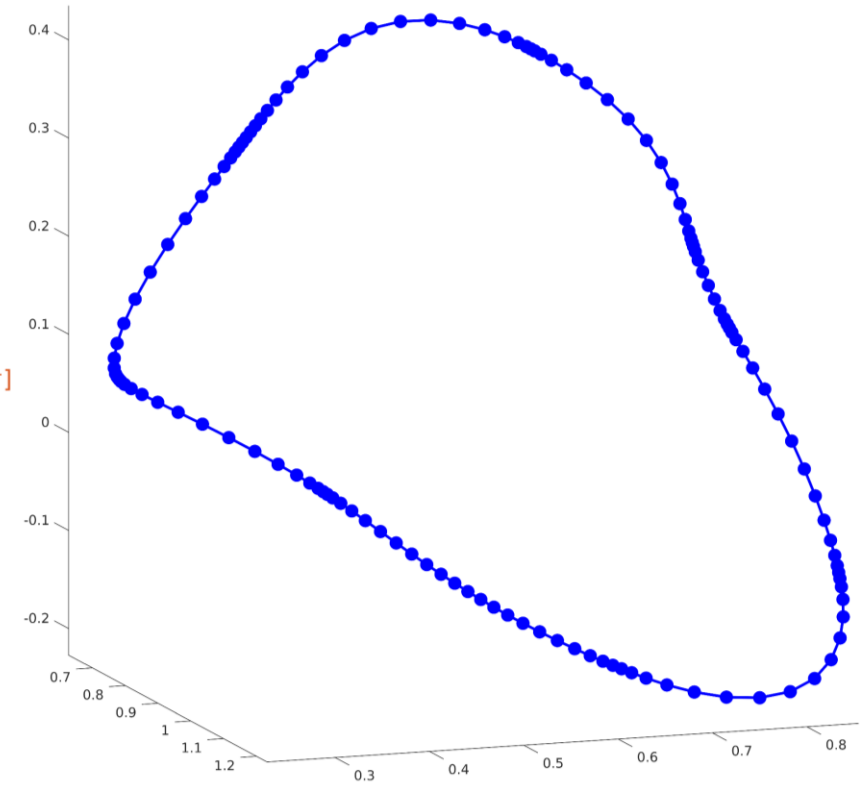
$$t \in [0, 2\pi)$$

- If objective functions are parameterization dependent, coils can move tangentially to minimize objective functions

Parameterization Dependent Objective Function

$$f_\kappa = \sum_{i=1}^{Ncoils} \int_0^{2\pi} \kappa_i^2 \, dt$$





Parameterization Independent Objective Function

$$f_\kappa = \sum_{i=1}^{Ncoils} \frac{1}{L_i} \int_0^{2\pi} \kappa_i^2 |\boldsymbol{r_i}| dt$$