BEAMS3D mostly agrees with ANTS for collisionless alpha confinement in Aten

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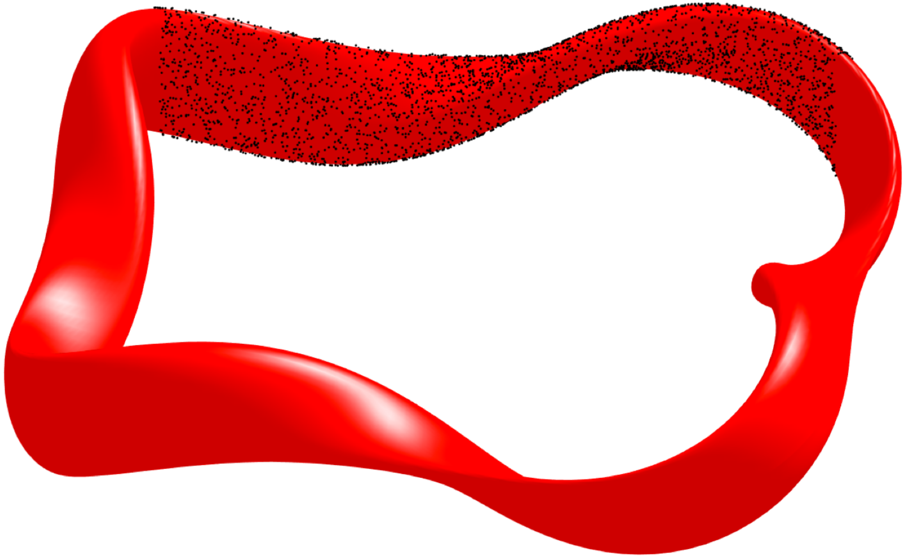
BEAMS3D code

- By Sam Lazerson.
- Part of the STELLOPT repository.
- Follows guiding centers in cylindrical coordinates.
- Can get **B** from VMEC, MGRID (coils), or both (virtual casing).
- MPI parallelization.
- Can do collisions (haven’t tried this).
- Documentation on the stellopt site and annotations in the Hdf5 output.
- Some regression tests exist.
- STELLOPT can target the BEAMS3D loss fraction.

New python script written to initialize particles in BEAMS3D

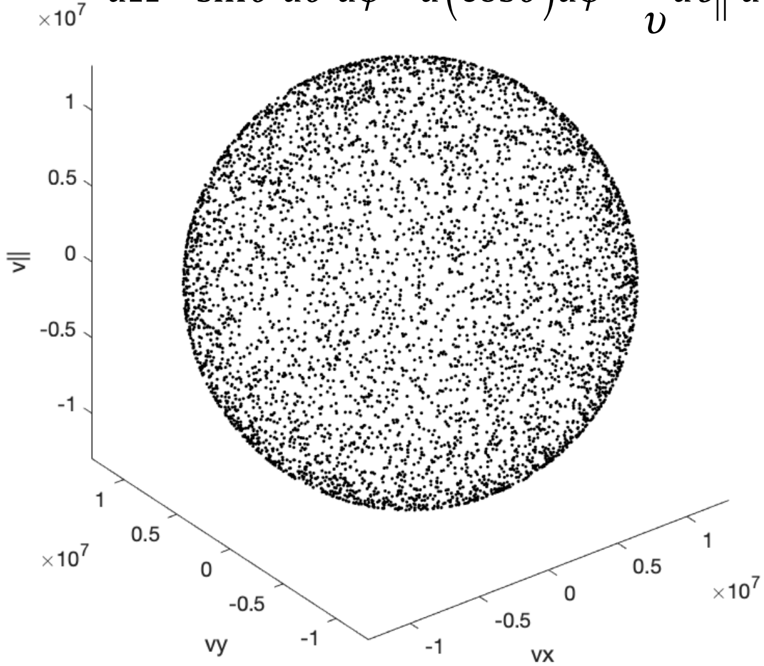
Position chosen to be evenly distributed in area, like alpha birth, not uniform in (θ, ζ).



Accounts for surface Jacobian $N = \left| \frac{\partial \mathbf{x}}{\partial \theta} \times \frac{\partial \mathbf{x}}{\partial \zeta} \right|$

Alpha birth is isotropic, so $v_{||}$ is a uniform random number in $[-v_\alpha, v_\alpha]$

$$d\Omega = \sin\theta \; d\theta \; d\varphi = d(\cos\theta) d\varphi = \frac{1}{\upsilon} d\upsilon_{||} \; d\varphi$$



(random gyrophase in $[0,2\pi]$ included in this figure)

BEAMS3D & ANTS use different guiding-center trajectory equations

BEAMS3D

Nonrelativistic

$$\frac{d\mathbf{x}}{dt} = v_{\|}\mathbf{b} + \frac{mv_{\|}^2}{qB}\mathbf{b}\times\left(\mathbf{b}\cdot\nabla\mathbf{b}\right) + \frac{mv_{\perp}^2}{2qB^2}\mathbf{b}\times\nabla B$$

$$\frac{dv_{\|}}{dt} = -\frac{\mu}{m}\mathbf{b}\cdot\nabla B$$

$$\mu = \frac{mv_{\perp}^2}{2B} = \text{constant}$$

Alphas are nonrelativistic: $v/c = 0.04 \ll 1$.
Codes should be similar when $\rho_* \ll 1$.

ANTS

Relativistic

D.V.Sivukhin,
in Reviews of Plasma Physics,
vol 1, pp.40-42

$$p_{\|} = mv_{\|}; \qquad p_{\perp} = mv_{\perp};$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v_{\|}^2 + v_{\perp}^2}{c^2}}};$$

$$a_{\|} = \frac{cp_{\|}}{eB}; \qquad a_{\perp} = \frac{cp_{\perp}}{eB}.$$

$$\mathbf{h} = \frac{\mathbf{B}}{B}$$

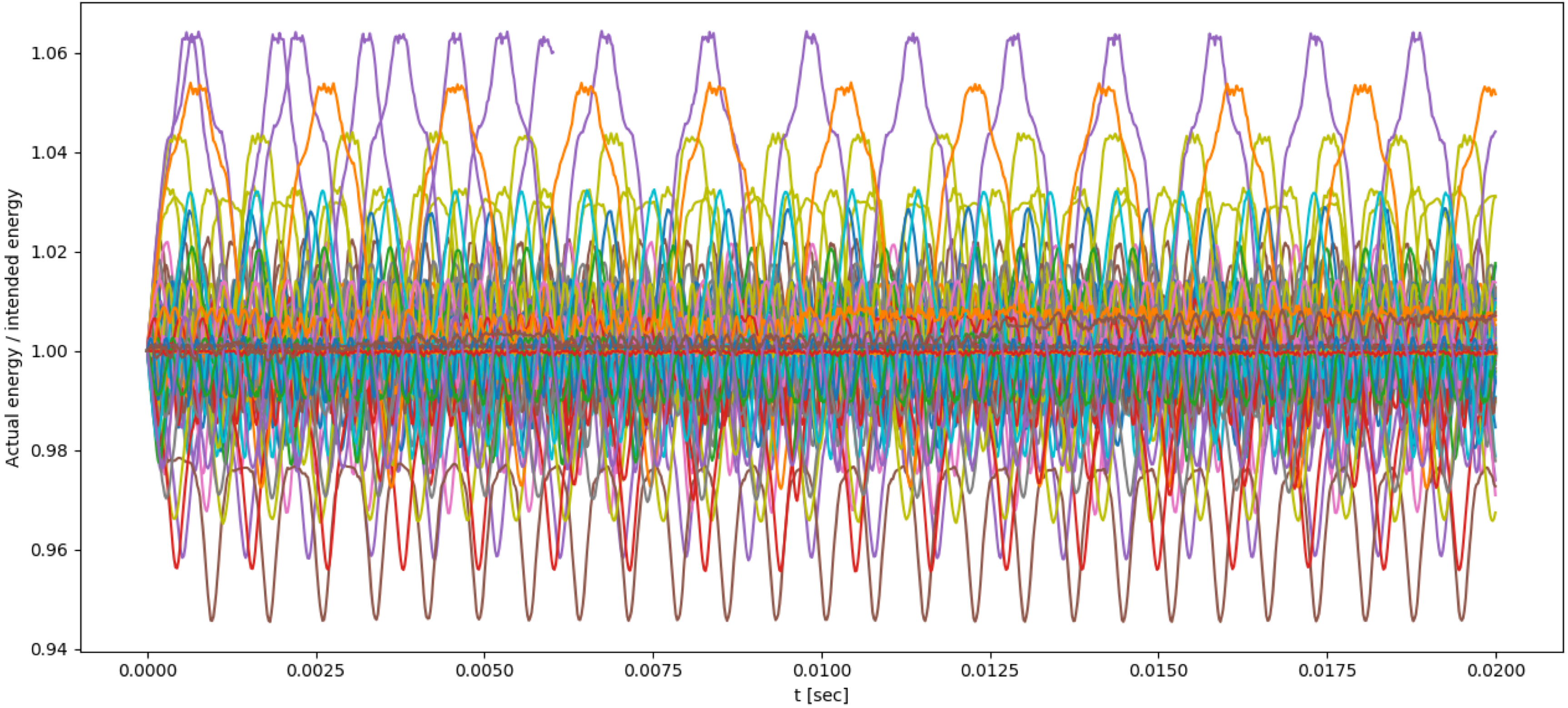
$$\dot{\mathbf{R}} = \left[v_{\|} + \frac{1}{2}v_{\perp}a_{\perp}(\mathbf{h}\,\mathrm{rot}\,\mathbf{h})\right]\mathbf{h} + \frac{c}{B^2}[\mathbf{EB}] + \frac{1}{2}v_{\perp}a_{\perp}\left[\mathbf{h}\,\frac{\nabla B}{B}\right] + v_{\|}a_{\|}\,[\mathbf{h}\cdot(\mathbf{h}\nabla)\,\mathbf{h}]; \qquad (6.1)$$

$$\dot{p}_{\|} = e(\mathbf{Eh}) + \frac{1}{2}v_{\perp}p_{\perp}\,\mathrm{div}\,\mathbf{h} + ea_{\|}\,(\mathbf{E}\,[\mathbf{h}\cdot(\mathbf{h}\nabla)\,\mathbf{h}]) - \frac{a_{\|}p_{\perp}v_{\perp}}{2}\left(\frac{\nabla B}{B}[\mathbf{h}\cdot(\mathbf{h}\nabla)\,\mathbf{h}]\right) - \frac{a_{\|}p_{\perp}v_{\perp}}{2}\,\mathbf{h}\,\mathrm{rot}\,(\mathbf{h}\nabla)\,\mathbf{h}; \qquad (6.2)$$

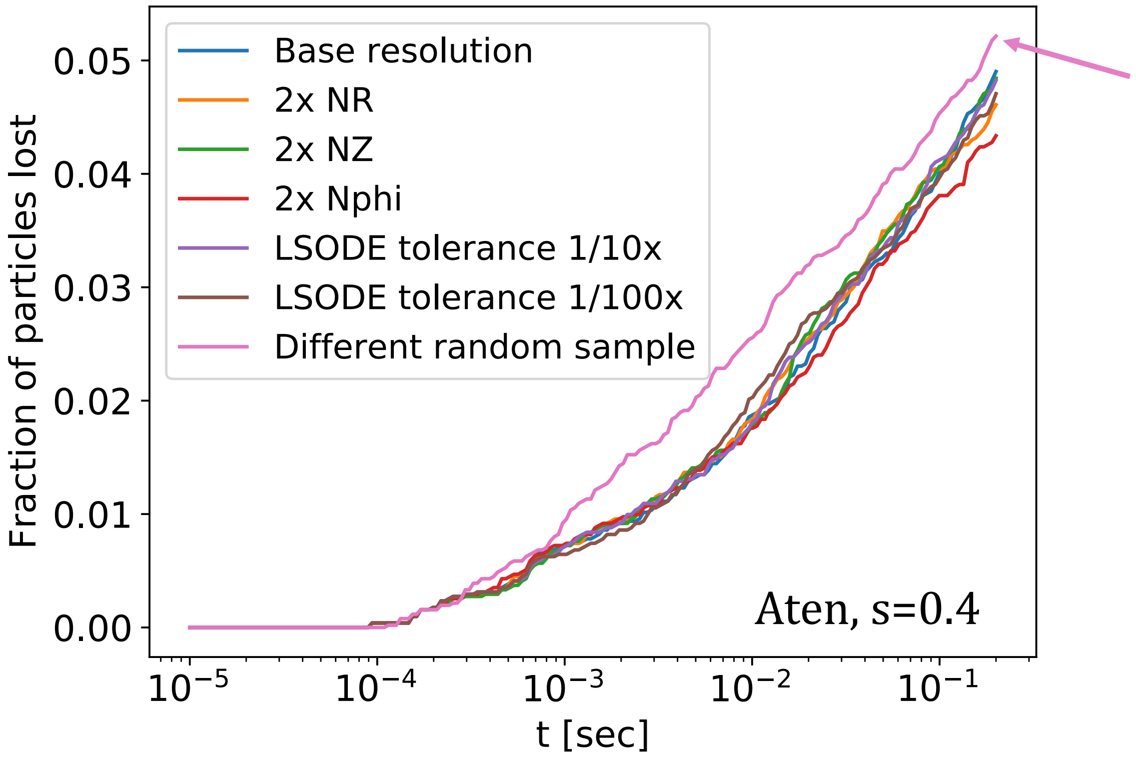
$$\dot{p}_{\perp} = -\frac{1}{2}v_{\perp}p_{\|}\,\mathrm{div}\,\mathbf{h} - \frac{ea_{\perp}}{2}(\mathbf{h}\,\mathrm{rot}\,\mathbf{E}) + \frac{ea_{\perp}}{2}\left(\mathbf{E}\left[\mathbf{h}\,\frac{\nabla B}{B}\right]\right) + \frac{ea_{\perp}}{2}(\mathbf{Eh})(\mathbf{h}\,\mathrm{rot}\,\mathbf{h}) + \frac{a_{\|}v_{\|}p_{\perp}}{2}\left(\frac{\nabla B}{B}[\mathbf{h}\cdot(\mathbf{h}\nabla)\,\mathbf{h}]\right) + \frac{1}{2}a_{\|}p_{\perp}v_{\|}\,\mathbf{h}\,\mathrm{rot}\,(\mathbf{h}\nabla)\,\mathbf{h}. \qquad (6.3)$$

In BEAMS3D equations, energy is not conserved exactly, though this may not be a problem

$$\frac{d}{dt}\left(\frac{mv^2}{2}\right) = \frac{mv_{||}^2}{2B}\frac{\mu}{qB}\mathbf{b}\times\left(\mathbf{b}\cdot\nabla\mathbf{b}\right)\cdot\nabla B \qquad \sim O\left(\rho_* \frac{mv^2}{2}\frac{v}{L}\right)$$



BEAMS3D convergence is checked by varying each numerical parameter



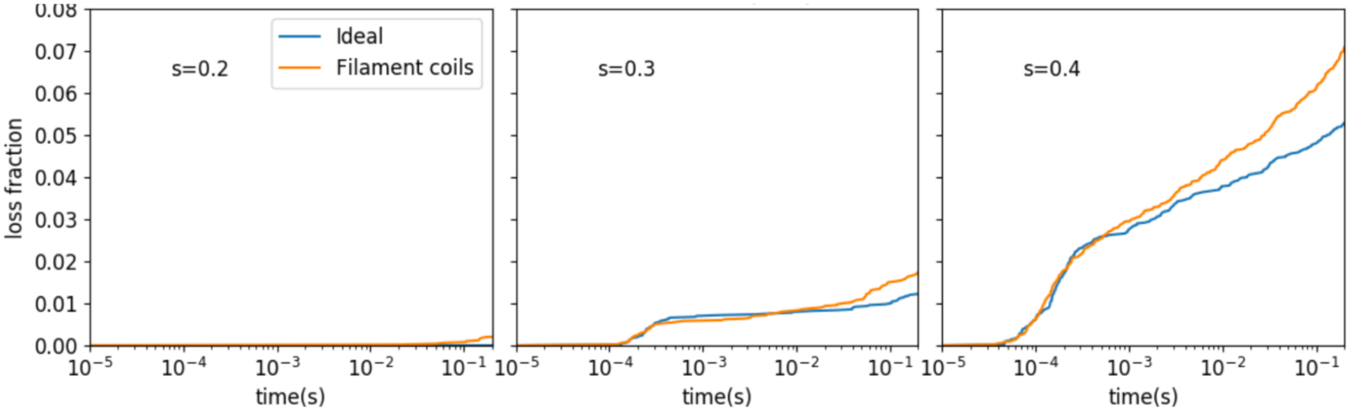
Same sampling algorithm & distribution, just a different initial seed, so the initial particle positions & velocities are different.

Base resolution: NR=NZ=Nphi=128, LSODE tolerance 1e-8, 5120 particles

ANTS vs BEAMS3D: Alpha losses as a function of surface are similar

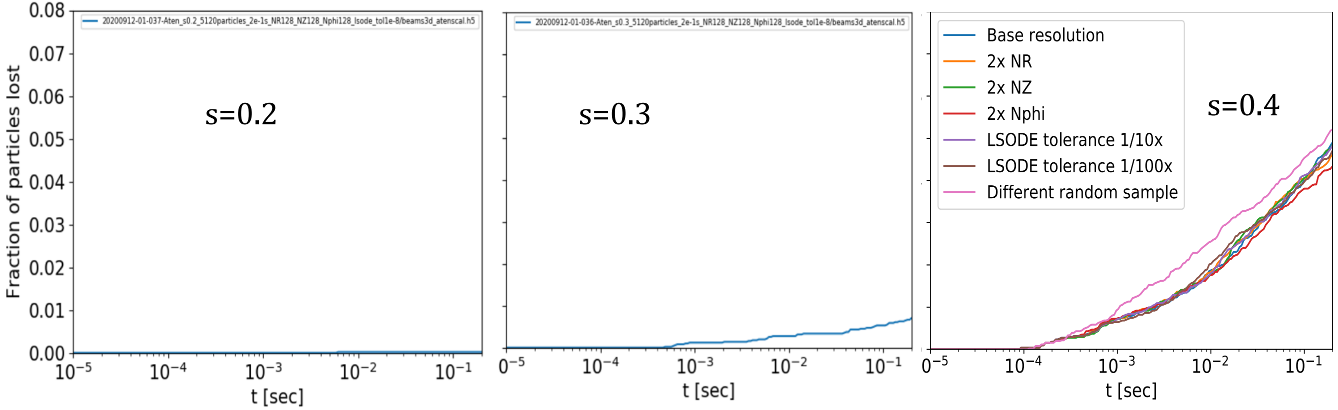
ANTS:
Figure 4 of
Bader et al, arXiv:2004.11426

5000 particles



BEAMS3D, using identical VMEC wout file:

(Only fixed-boundary case considered, not one with coils.)

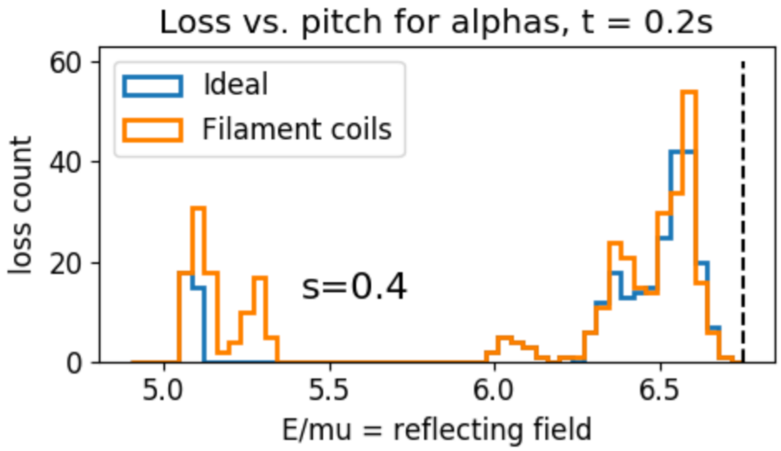


Losses by 0.2 s are similar, and very small for a stellarator. ANTS shows more losses at early times, 10^{-4}-10^{-3} s. Difference probably explainable by the different guiding-center equations?

ANTS vs BEAMS3D: Lost pitch angles are similar

ANTS:
Figure 5 of
Bader et al, arXiv:2004.11426

5000 particles



BEAMS3D, using identical VMEC wout file:

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