Ideal Ballooning Stability optimization through profile shaping in STELLOPT

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Motivation

- finite beta optimization thus far has focused on improving energetic particle confinement and rotational transform profiles
- MHD stability remains an issue – particularly pressure driven modes as we push to finite beta configurations
- can profile shaping gain a stability advantage for ideal MHD ballooning modes?

Outline

- configuration and starting profiles
- optimization targets and variables
- BOOTSJ → SFINCS optimization
- SFINCS only optimization

Configuration and profiles

- Wistell-A configuration (ATEN)
    - scaled to 2.5T
    - free boundary VMEC
    - flux surface shaping held constant during optimization
- Te = 3.5keV * (1-s)
    - Ti = Te; ion root, ambipolar E_r solution for bootstrap currents in SFINCS
    - specified as akima spline
- $n_e = n_i =$ 9e19 m^{-3} * $(1-s^5)$, fixed throughout optimization

Optimization in STELLOPT

Targets:

- volume avg. β
- ideal MHD ballooning growth rates evaluated with COBRAVMEC provide stability metric

$$\chi^2 = \frac{\left(f_\beta^{target} - f_\beta^{equilibria}\right)^2}{\sigma_\beta^2} + \sum_{s=0}^{1}\sum_{\theta=0}^{n}\sum_{\zeta=0}^{m} \frac{\left(f_{s\theta\zeta}^{target} - f_{s\theta\zeta}^{equilibria}\right)^2}{\sigma_{s\theta\zeta}^2}$$

(71 surfaces -2)*(5 poloidal)*(5 toroidal) = 1725 stability targets

Variables:

- temperature (electron and ion) spline knots
- knots varied in vertical direction, fixed in radial direction

1.97% initial β, BOOTSJ → SFINCS

weights with BOOTSJ:

- stability at all flux surfaces with equal weighting, σ = 1e-2
- volume avg β, σ = 1e-4

TE/TI spline:

- specified values at s = [0.0, 0.2, 0.4, 0.6, 0.8]
- found best results with 3 variables

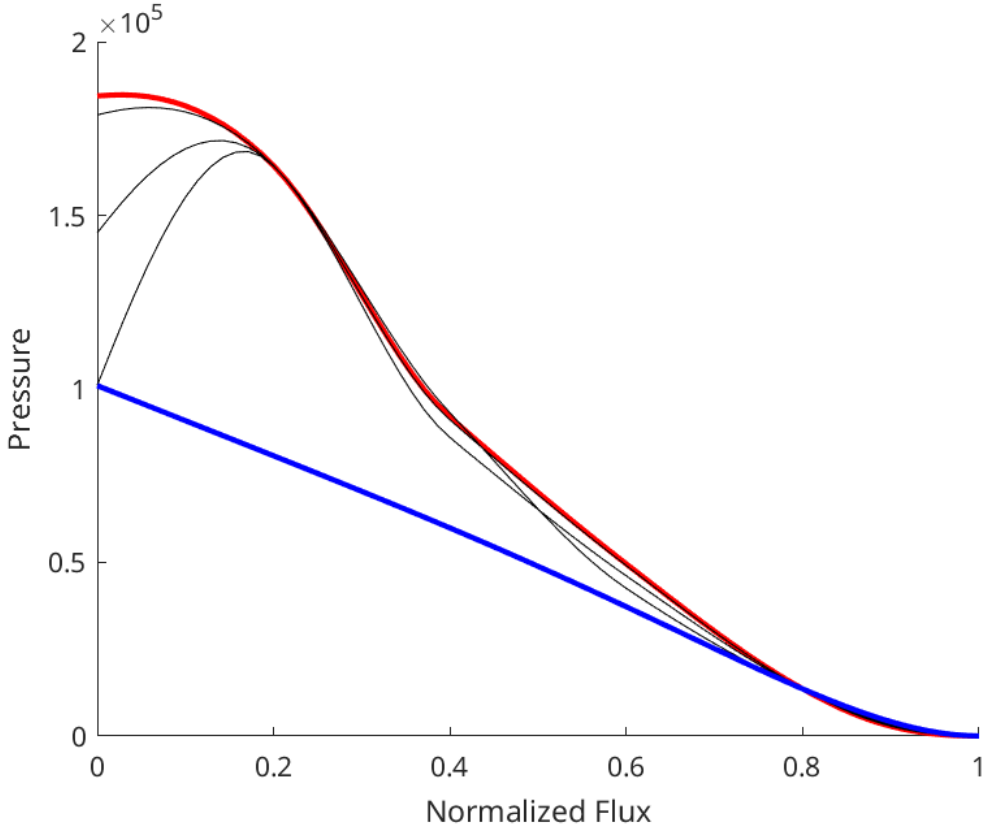
procedure:

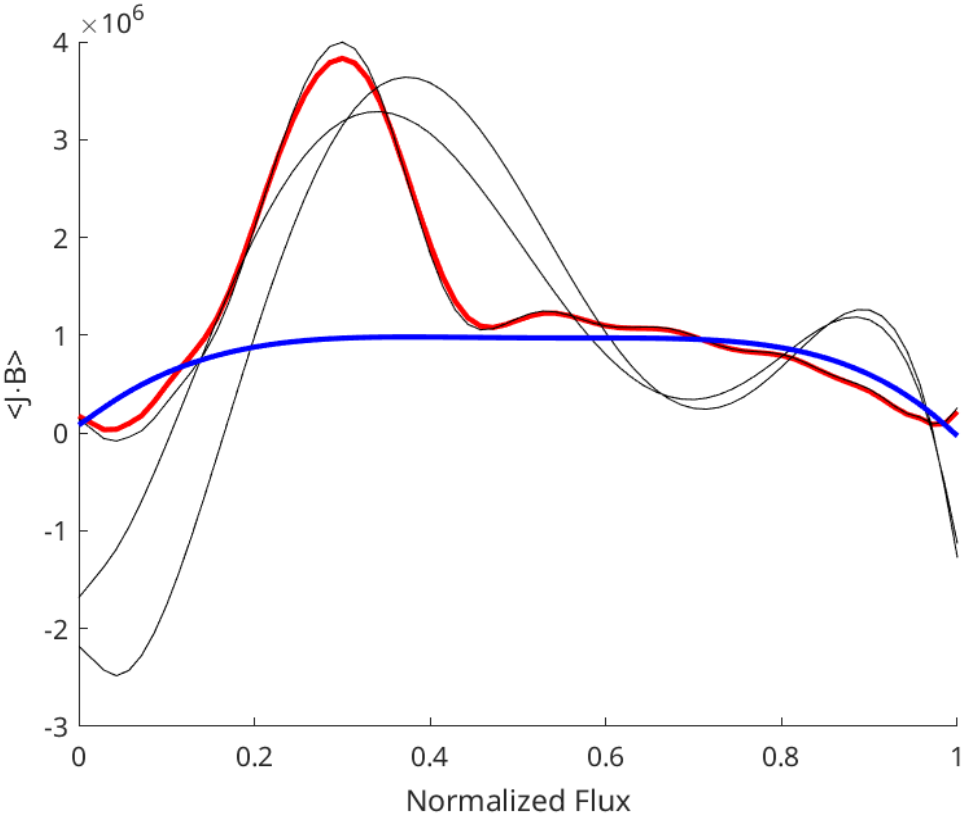
- push beta up with stronger relative weighting
  - calculate bootstrap current with BOOTSJ iteratively to provide self-consistent J_{BS} during optimization (vboot loop)
- after series of optimization loops, recalculate current with SFINCS
- flip relative weighting to emphasize stability and optimize at fixed J

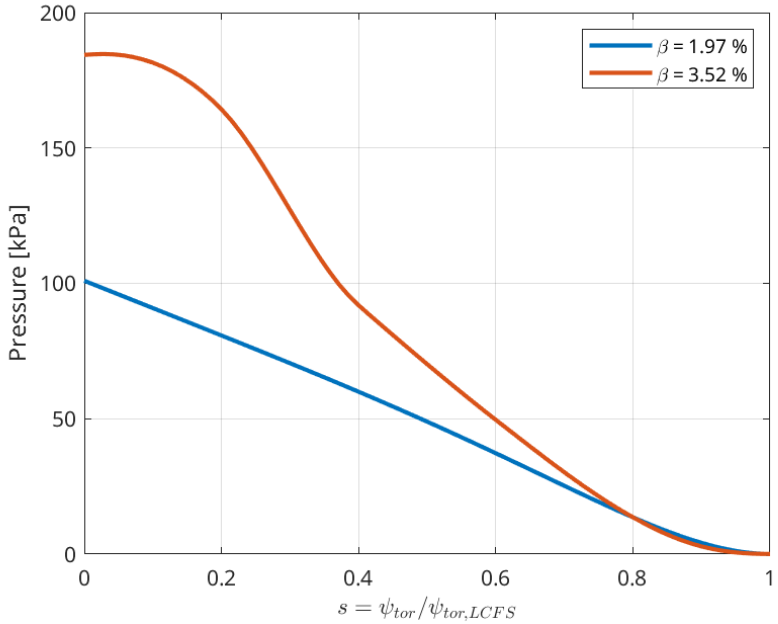
BOOTSJ

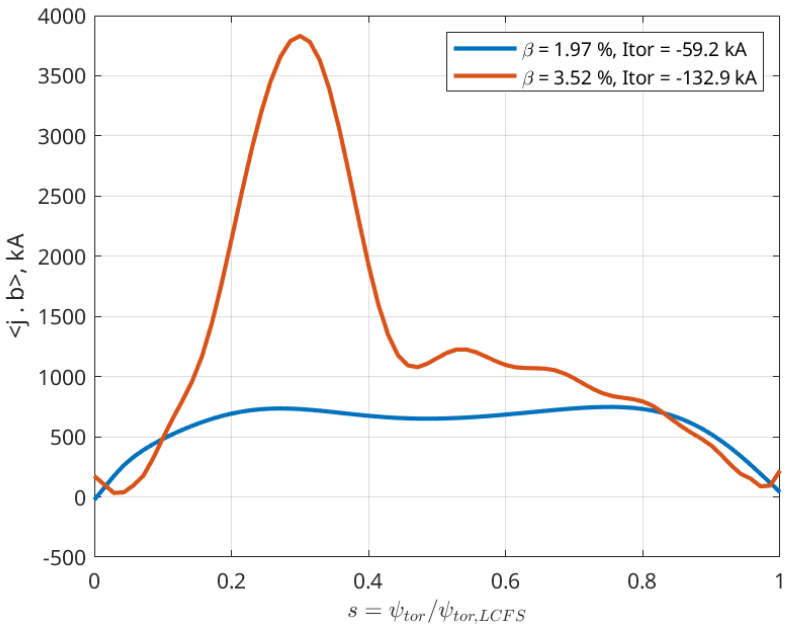
- vary any 3 of 0.0, 0.2, 0.4, 0.6 in series of optimization loops (each black curve is end result of a separate STELLOPT run)
    - increase β
    - forced monotonic pressure manually (via choice of variables)

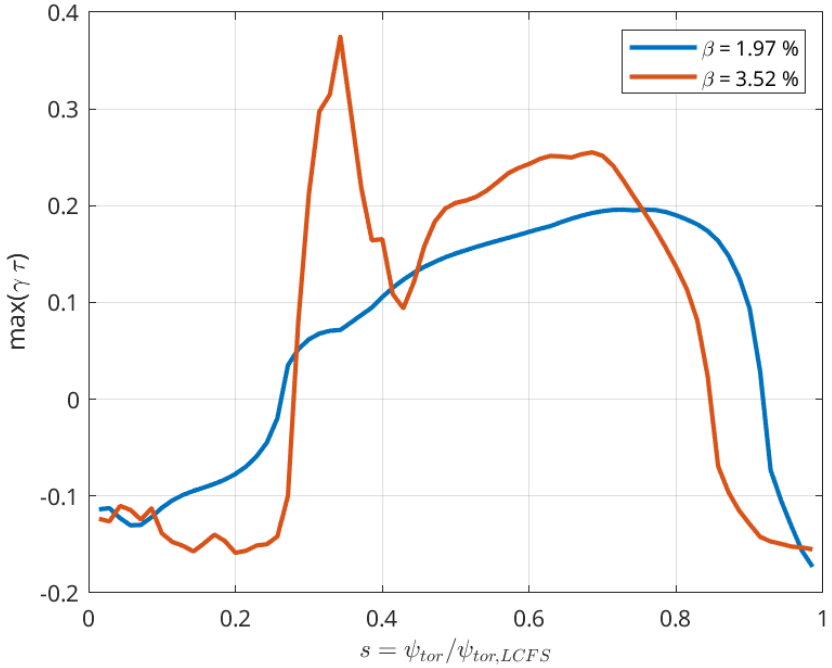
- at red solution, temperature profiles do not move under optimization regardless of weighting

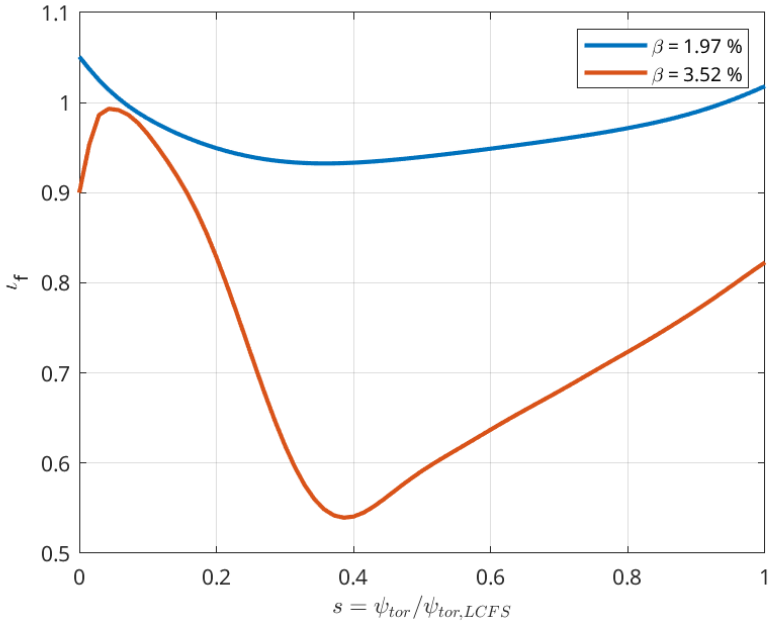




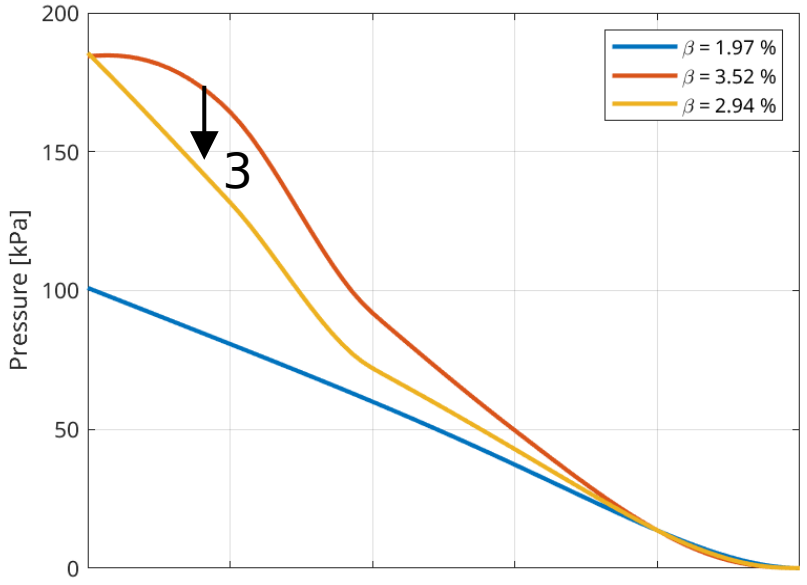


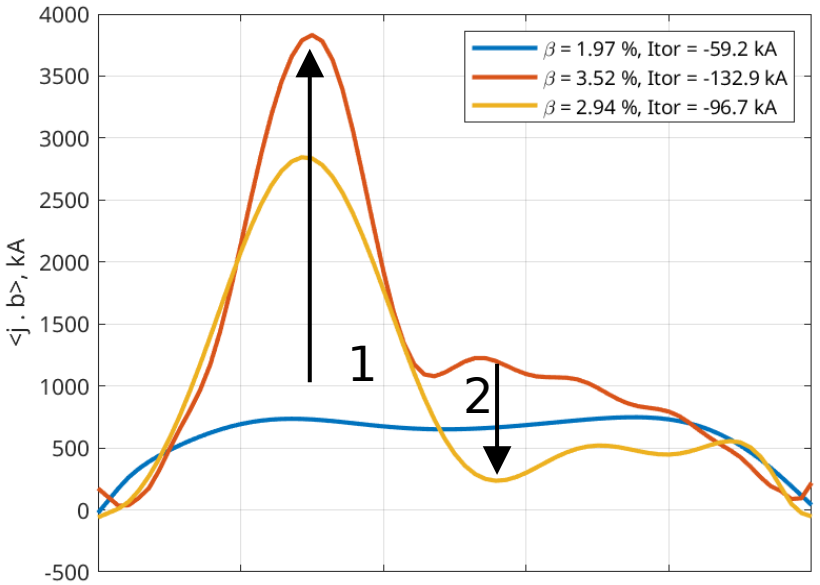




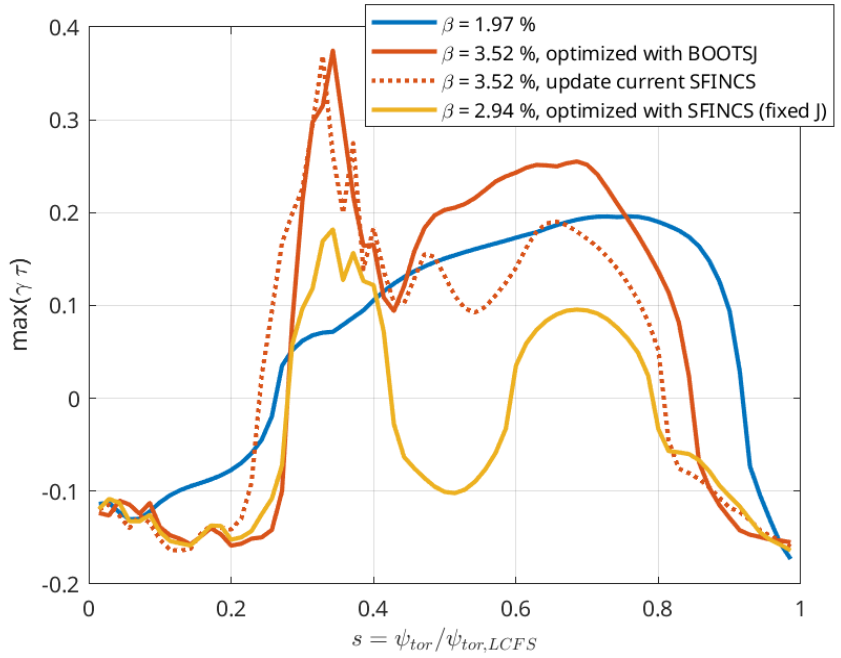


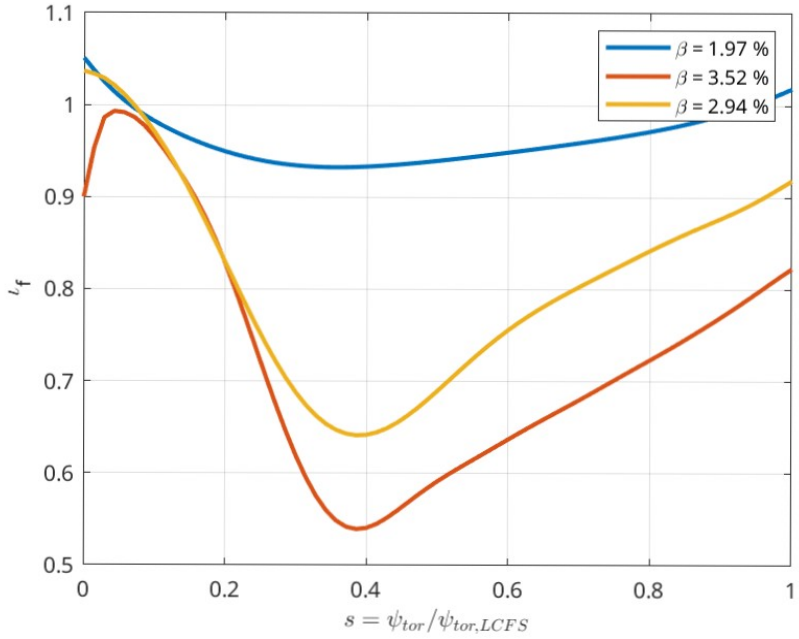
- growth rates pushed towards core – but not improved
- issues with BOOTSJ giving reliable currents and rotational transform near core
- from here switched to optimization with fixed SFINCS currents and flip weights





Sfincs unopt.
Bootsj opt.
Sfincs opt.

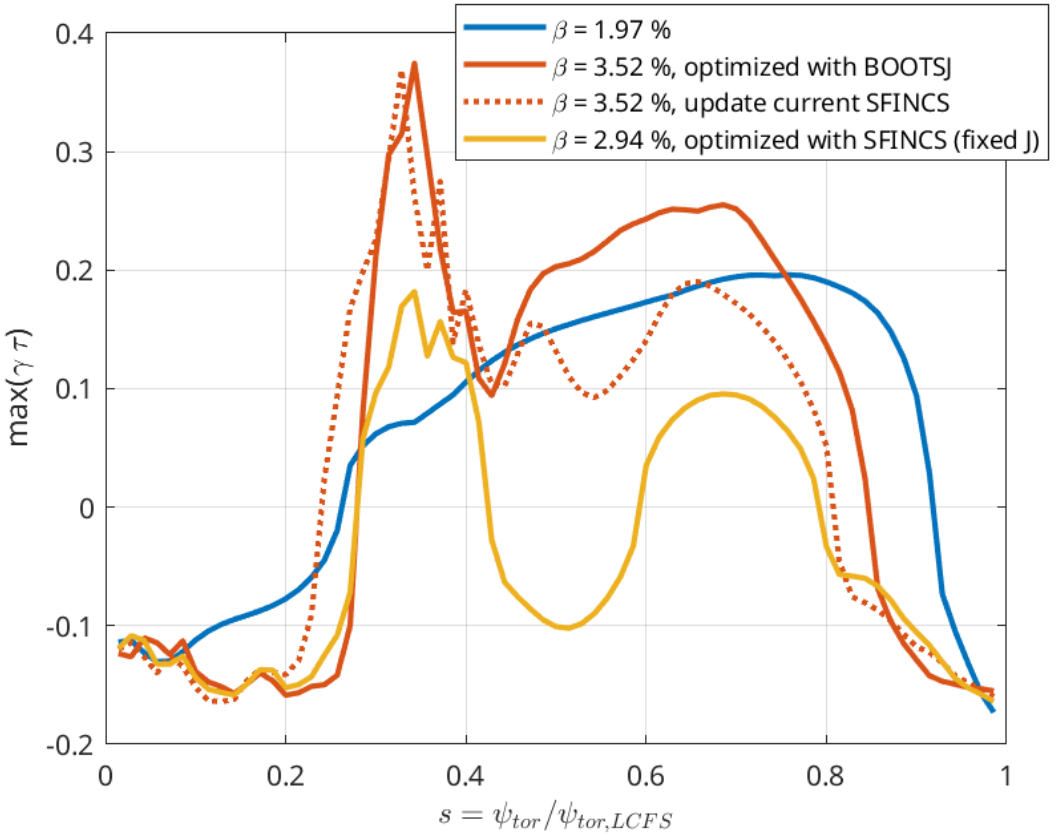


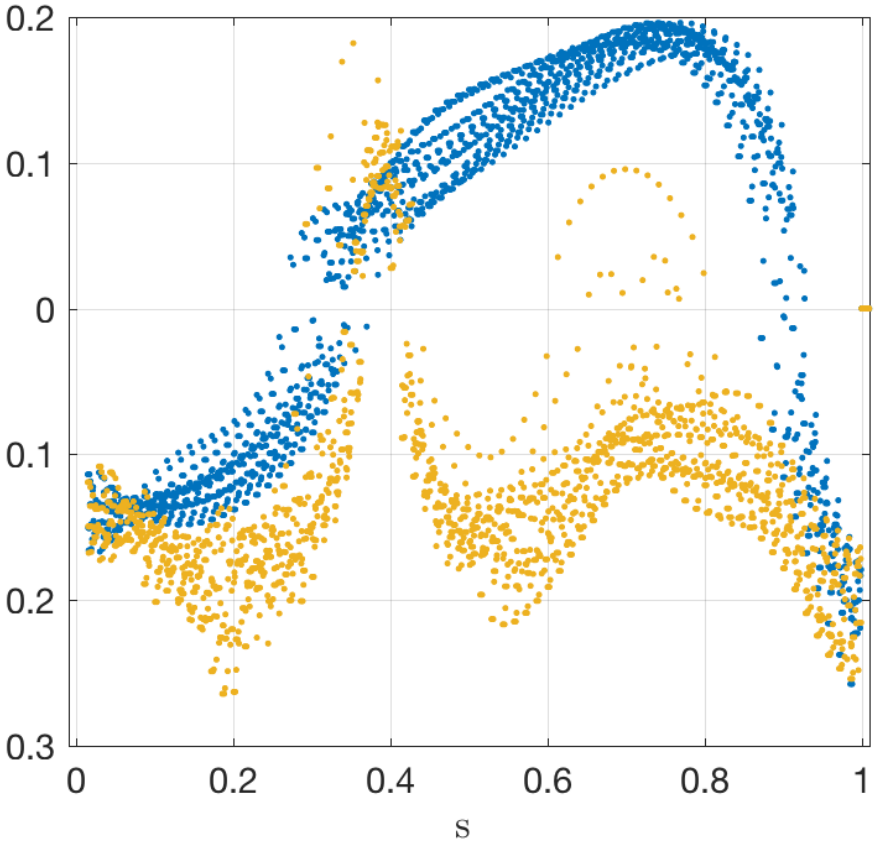


1. optimize profiles in STELLOPT, with BOOTSJ current updates in optimization loops
2. update current with SFINCS at fixed pressure profile
3. adjust weights, optimize profiles in STELLOPT at fixed SFINCS current

- initial to final (blue to solid yellow) – fastest growing ideal ballooning mode decreases, while beta increases – for s > 0.5 most postive growth rates eliminated

- however, ballooning growth rates remain positive in some regions – small region around s = 0.3-0.4 unstable





Notes for future work:

- profile monotonicity during temperature spline optimization is not currently enforced
- my splines specified with 5 knots, and found best result when any 3 are varied at once – increasing # of variables in future could be useful
- well converged SFINCS currents difficult to calculate for some profiles

1.97% initial β, fixed SFINCS current

weights:

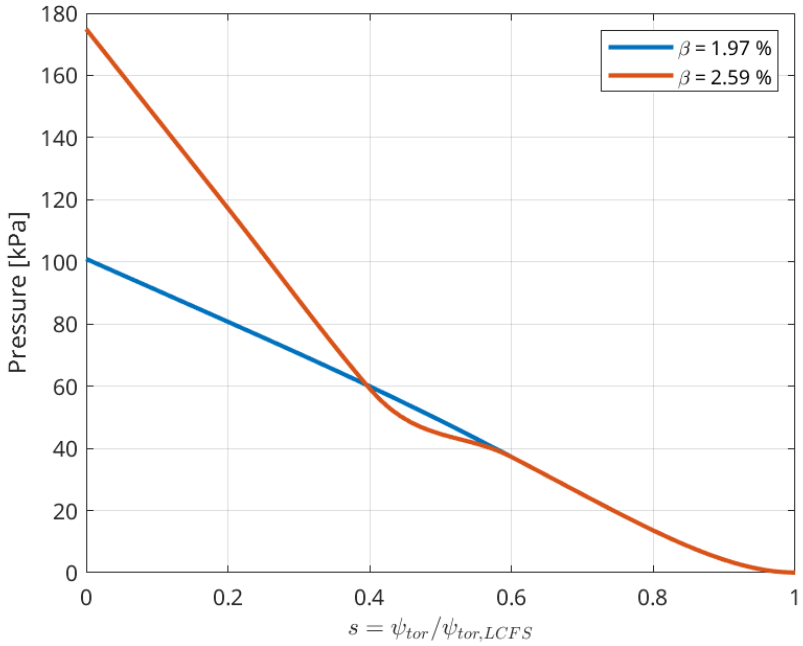
- stability at all flux surfaces with equal weighting, σ = 1e-4
- volume avg β, σ = 1e-1

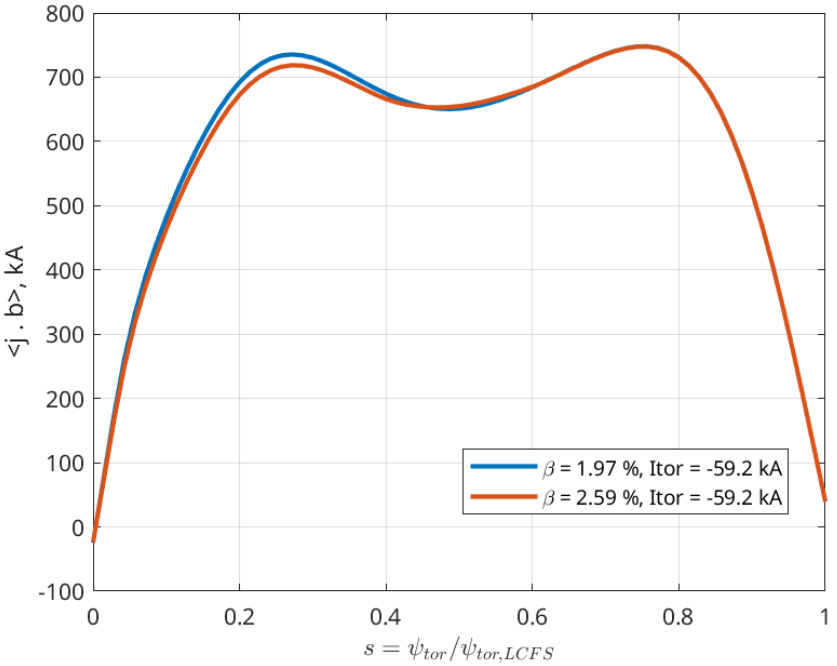
TE/TI spline:

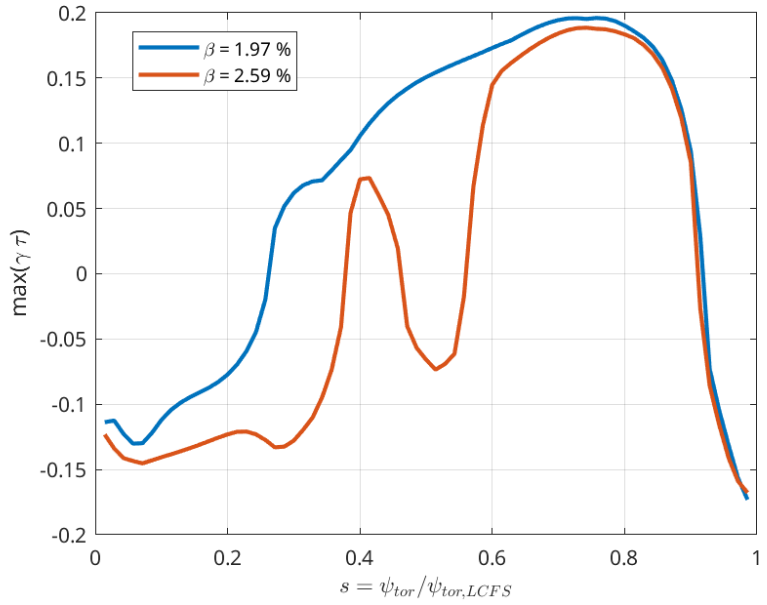
- specified values at s = [0.0, 0.2, 0.4, 0.6, 0.8]
- found best results with 3 variables

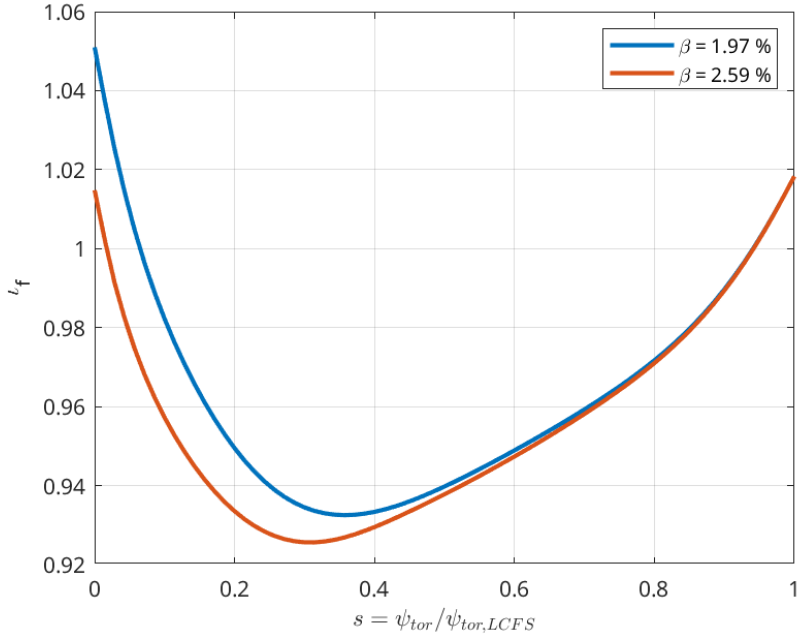
procedure:

- optimize for stability with strong relative weighting
- keep high beta target with weak weighting to prevent profile from collapsing towards zero (trivial stabilization)
- optimize in STELLOPT with fixed current calculated with SFINCS, try to have small changes in profiles

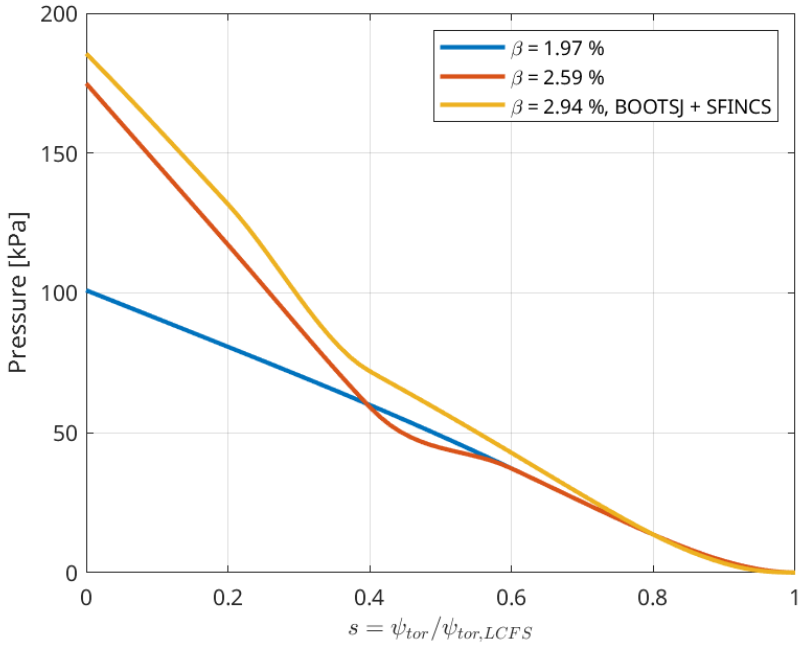


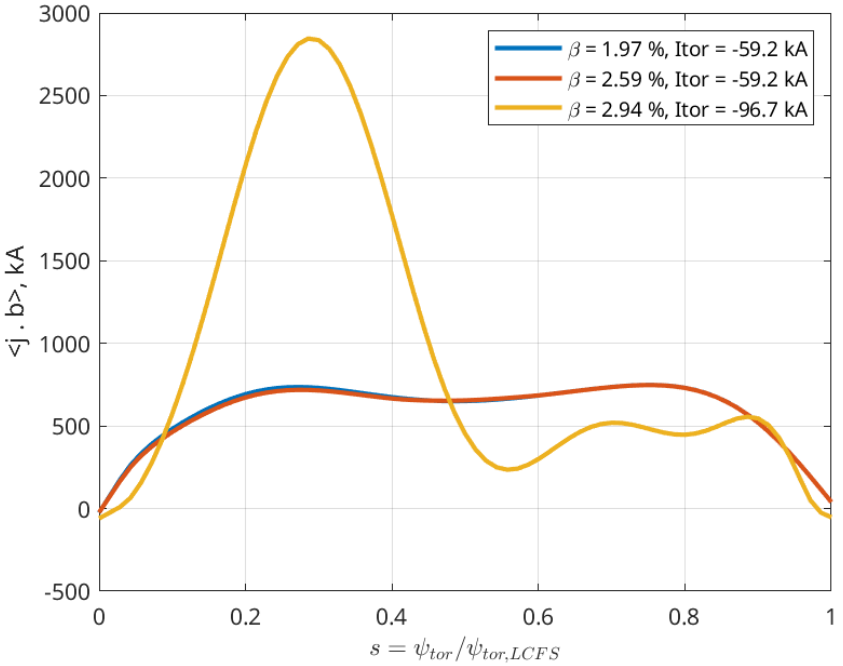


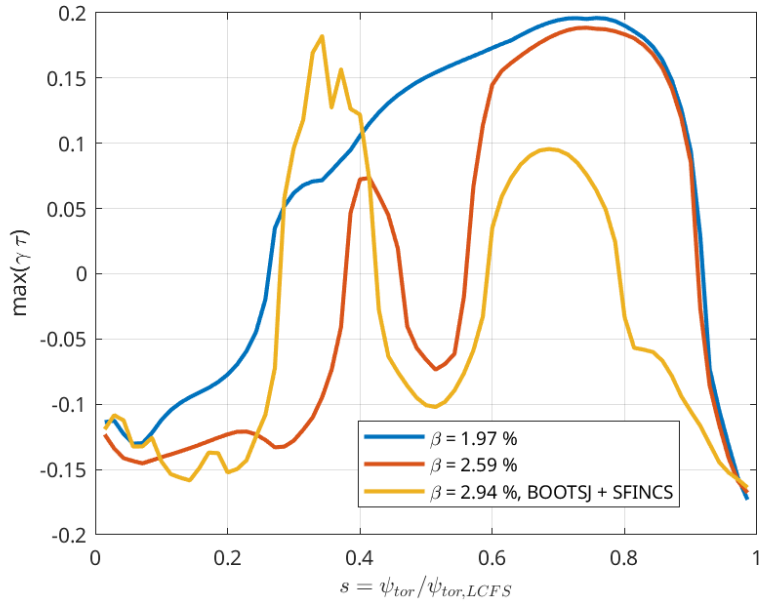


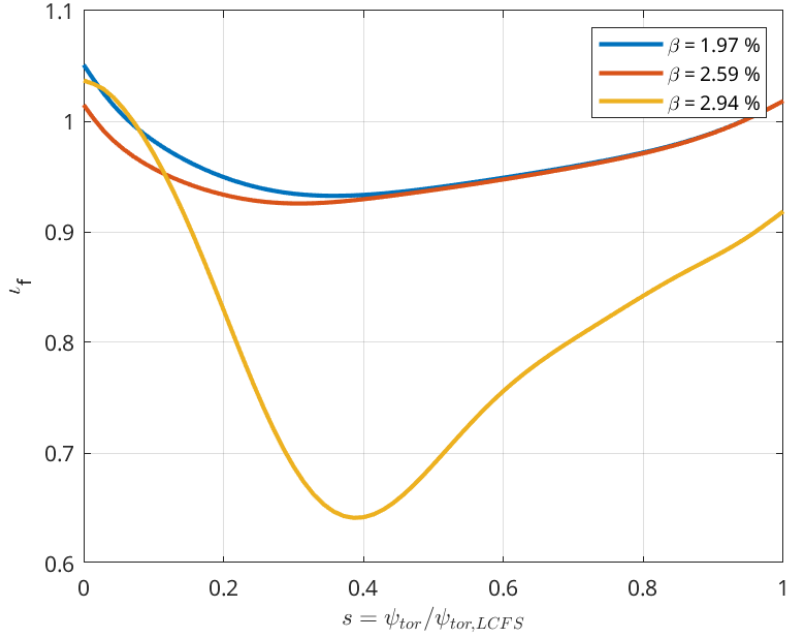


- results after single optimization loop shown below, where temperature spline knots varied at s = 0.0, 0.2, and 0.4



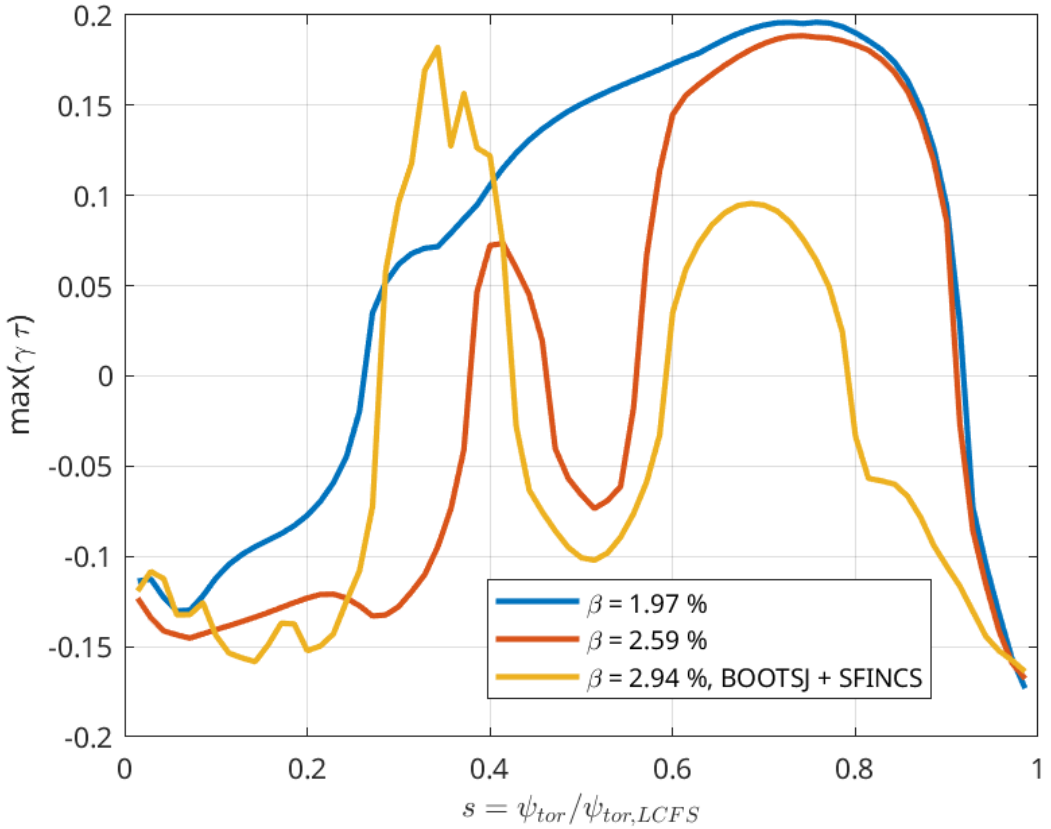


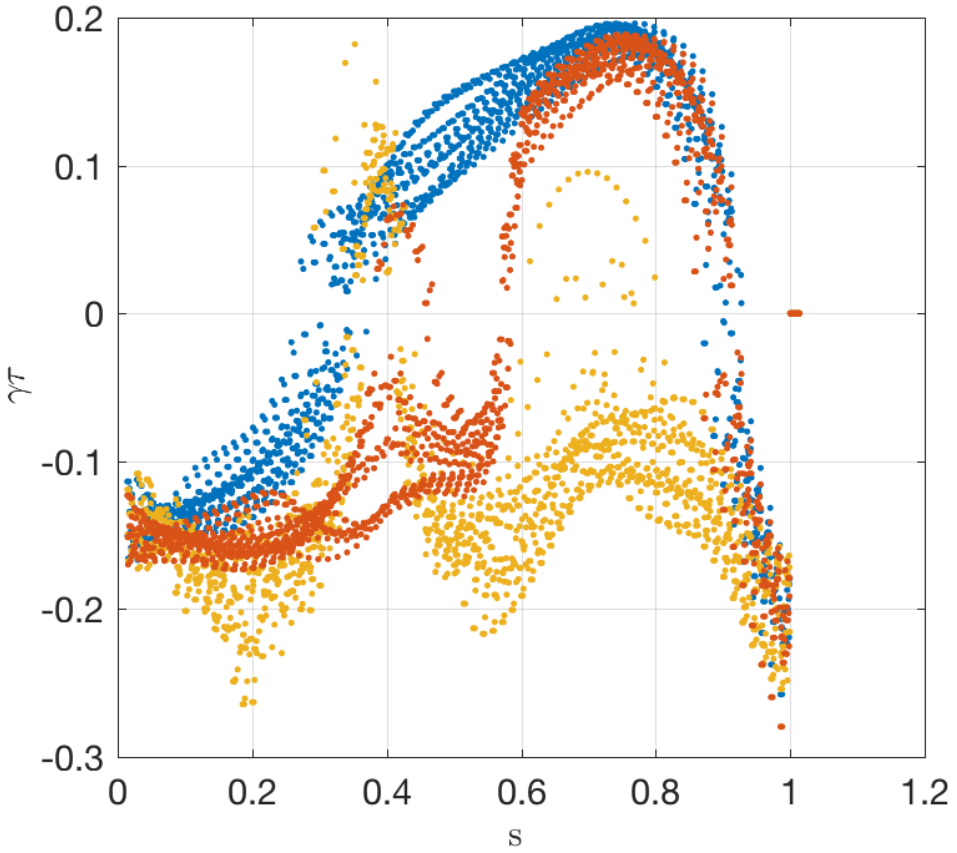




- results after single optimization loop shown below, where temperature spline knots varied at s = 0.0, 0.2, and 0.4

- similar to before, optimization with fixed SFINCS currents yields higher beta at reduced max ideal ballooning growth rates
- In 0.5<s<0.7 greater fraction of surface unstable in red case compared to yellow
- expect a difference in red growth rates from current update





Observations

- shaping alone can be used to change where in radial location growth rates peak
    - not shown here, but a sufficiently peaked temp. profile can result in ballooning growth rates deep in plasma core – i.e. if edge stability is desired
- optimizing spline profiles without a β target results in knots moving towards zero (or minimum constraint)
    - finding “sweet spot” in relative weighting between β target and stability is important
    - I focused on a β target higher than current configuration value, holding β constant with high weight might be useful

Conclusions / Future Work

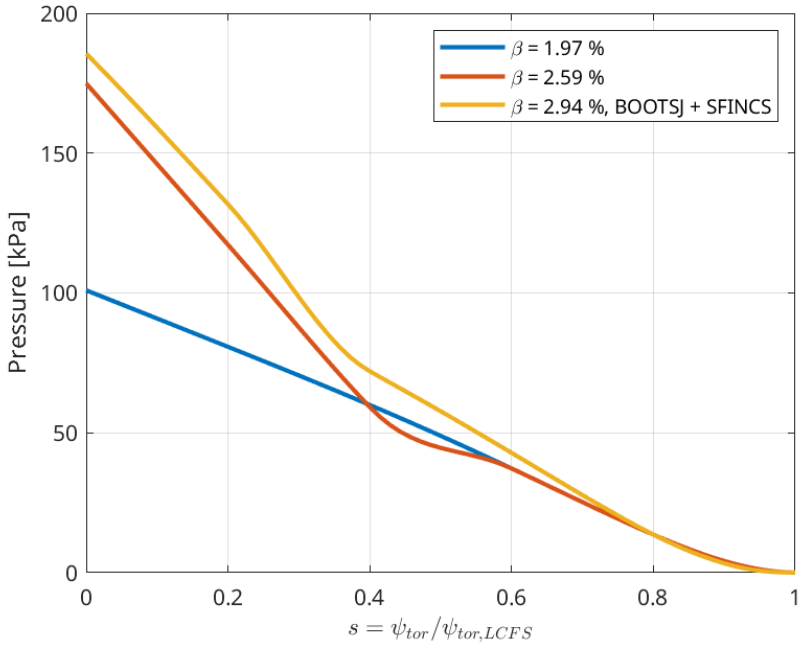
- optimizing temperature profiles for shape and magnitude can reduce **max** ideal ballooning growth rates – also can reduce fraction of a surface that is susceptible to unstable modes

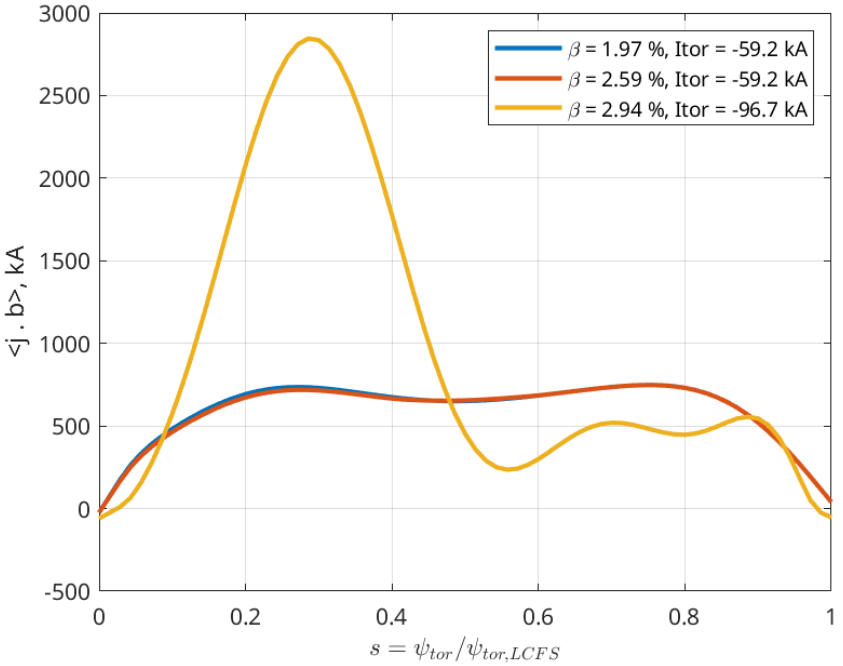
- changing the magnitude (i.e. plasma beta) appears to be the best way to reduce total instability over plasma volume

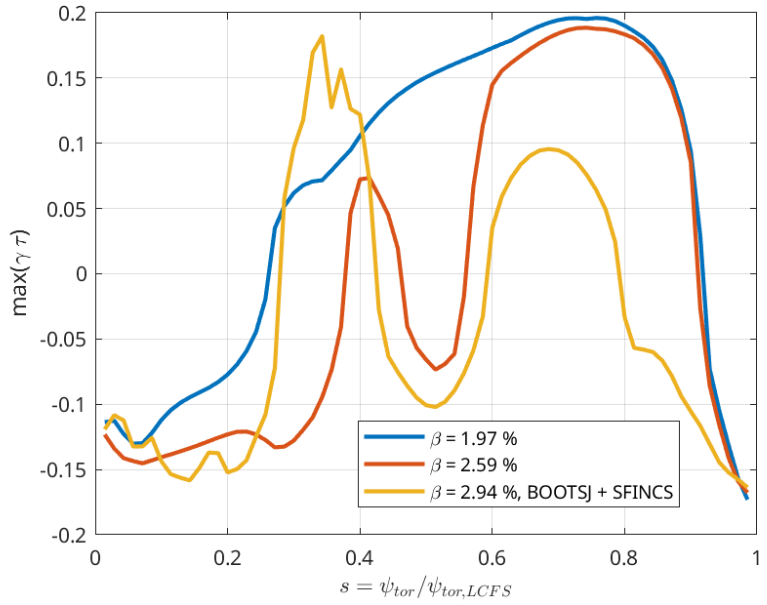
- can this stability optimization process be automated?
    - enforcement of monotonic profiles
    - increase number of variables to reduce need for user changes
    - include additional targets, i.e. energetic particle confinement, iota – profile opt. or a subsequent boundary optimization

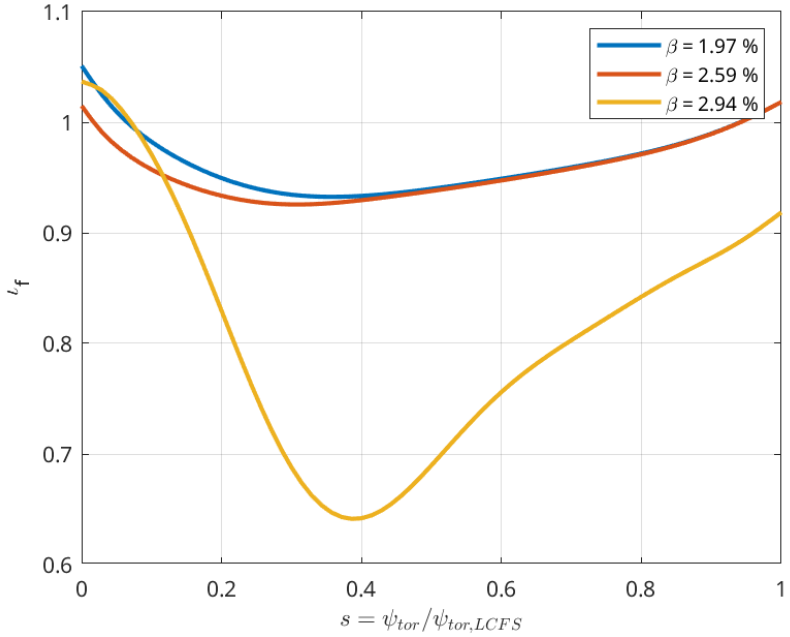
Backup slides

- The following slides contain contour plots of the growth rates at several radii
  - VMEC coordinate: s = 0.3, 0.5, 0.7
  - Single field period, theta = poloidal angle, zeta = toroidal angle

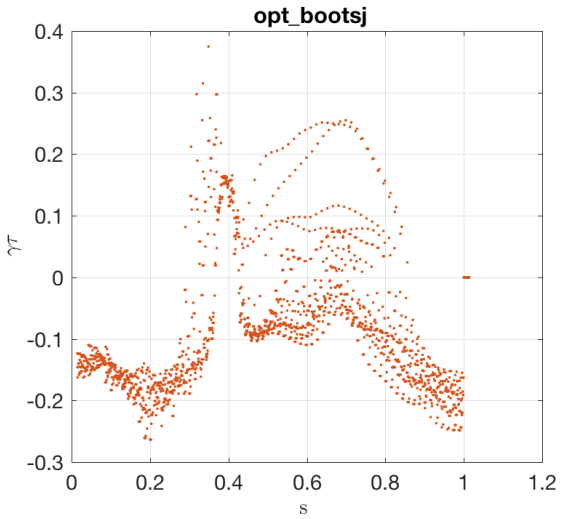

Pressure [kPa]
$s = \psi_{tor}/\psi_{tor,LCFS}$
β = 1.97 %
β = 2.59 %
β = 2.94 %, BOOTSJ + SFINCS

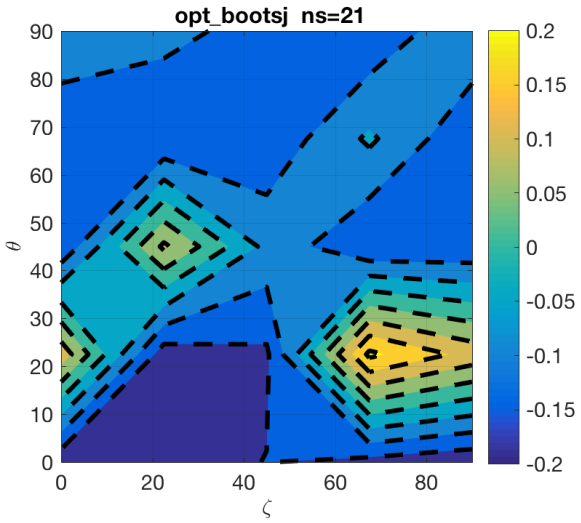

<j . b>, kA
$s = \psi_{tor}/\psi_{tor,LCFS}$
β = 1.97 %, Itor = -59.2 kA
β = 2.59 %, Itor = -59.2 kA
β = 2.94 %, Itor = -96.7 kA

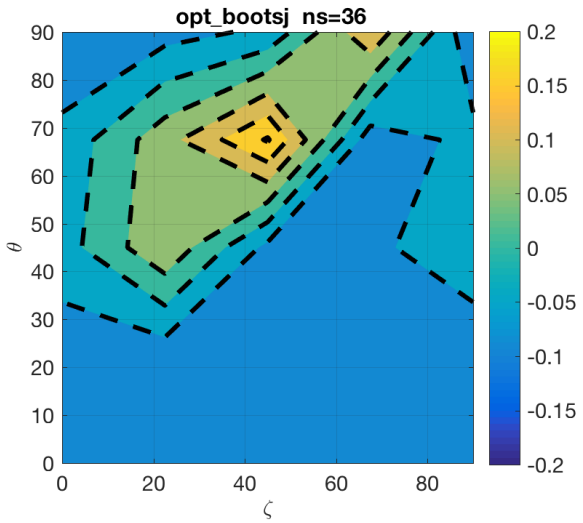

max(γ τ)
$s = \psi_{tor}/\psi_{tor,LCFS}$
β = 1.97 %
β = 2.59 %
β = 2.94 %, BOOTSJ + SFINCS

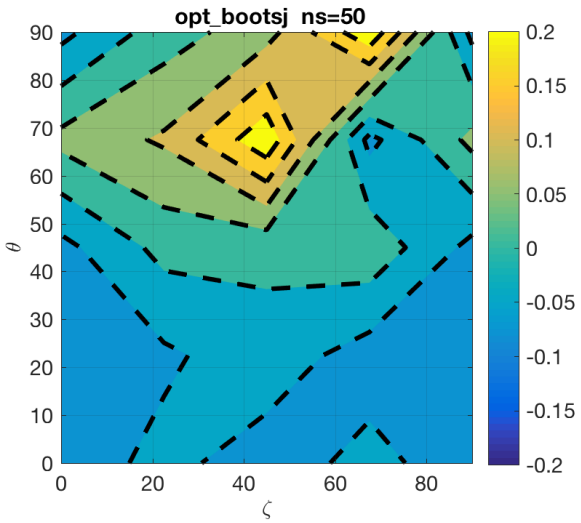

ι_f
$s = \psi_{tor}/\psi_{tor,LCFS}$
β = 1.97 %
β = 2.59 %
β = 2.94 %

Beta = 3.52, vboot(bootsj) opt

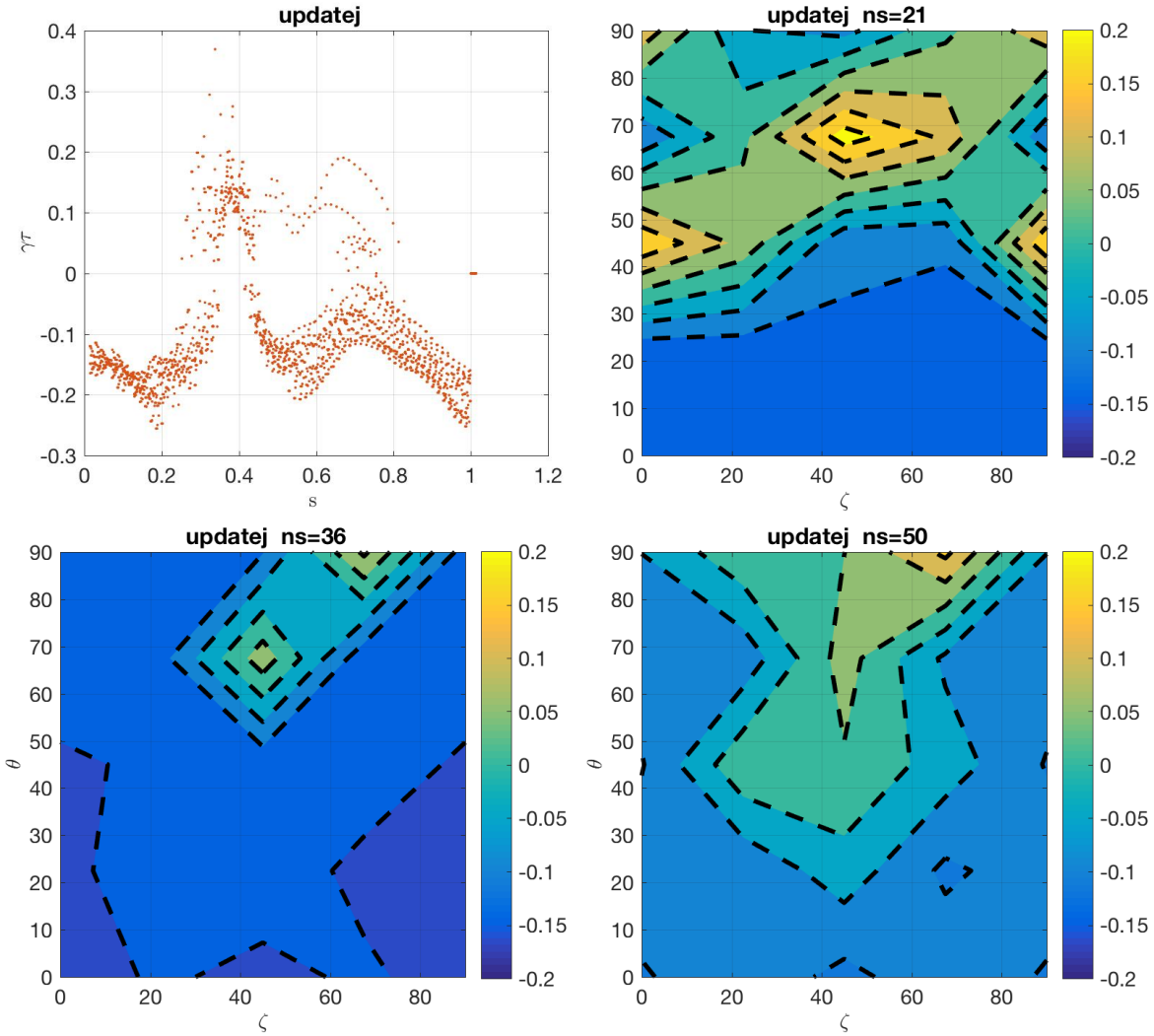




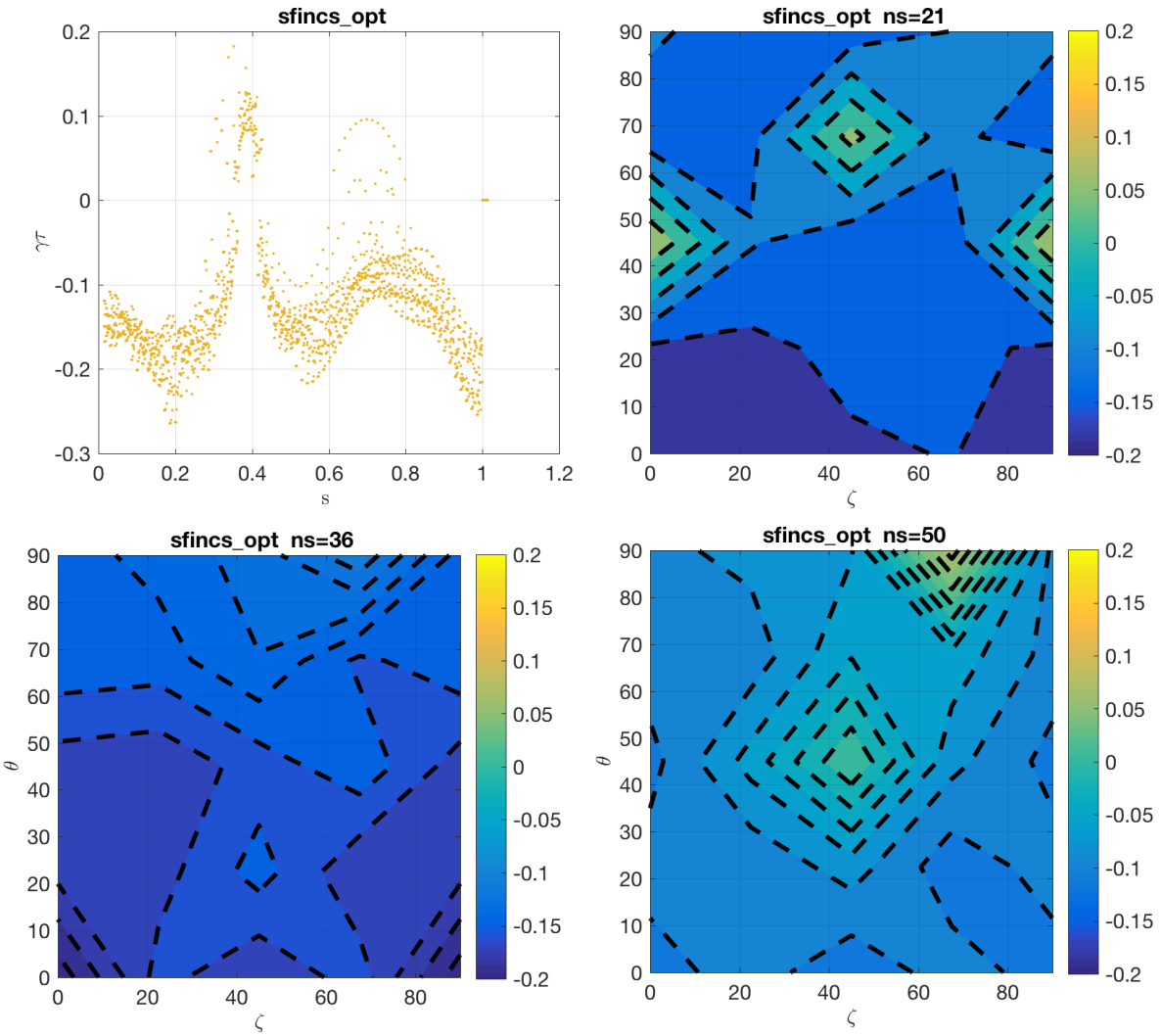




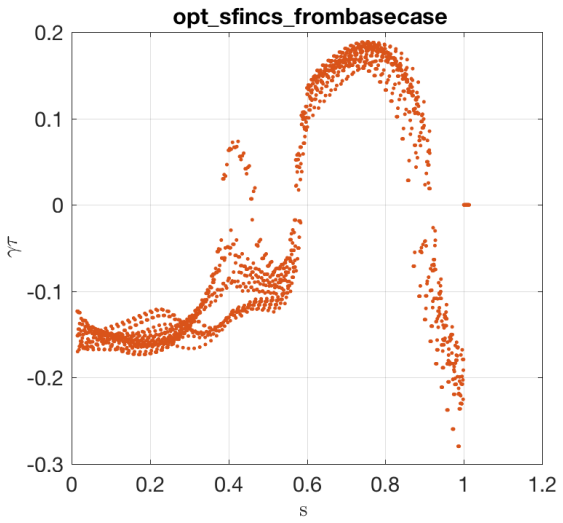
Beta=3.52, re-calc bootstrap current with SFINCS

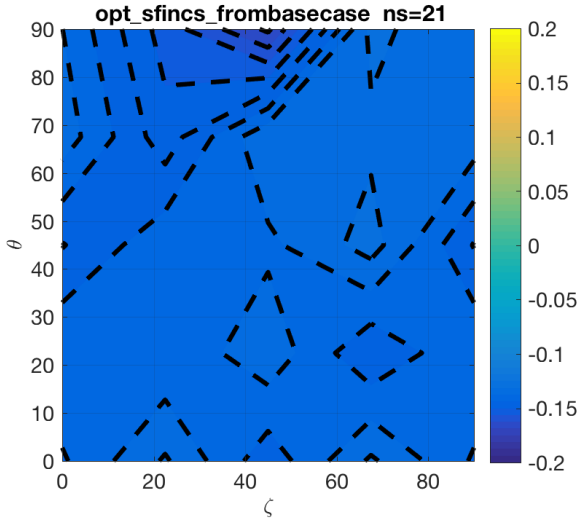


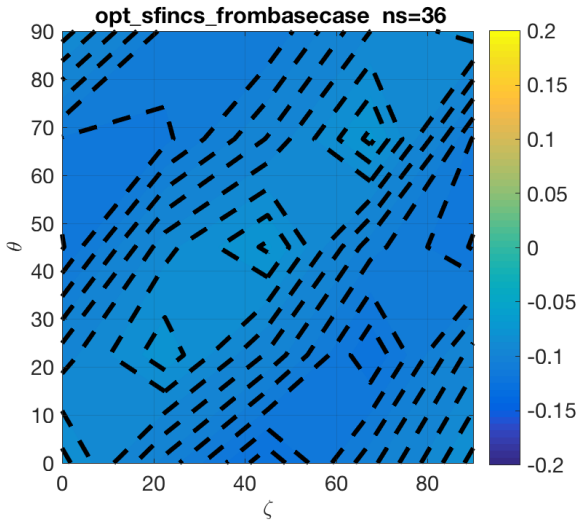
Beta=2.94, Optimization with bootstrap current fixed at latest sfincs update (previous slide).

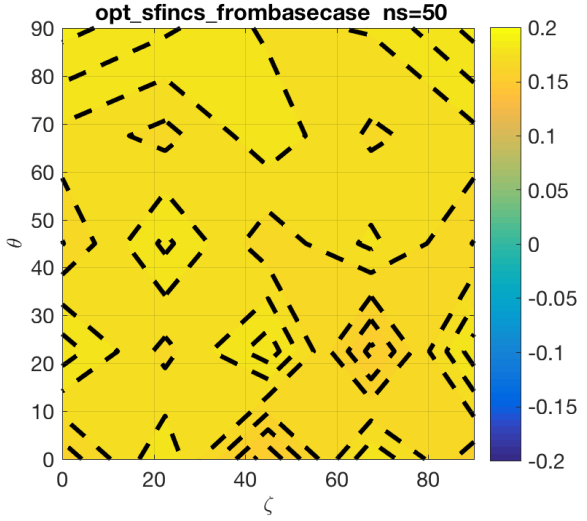


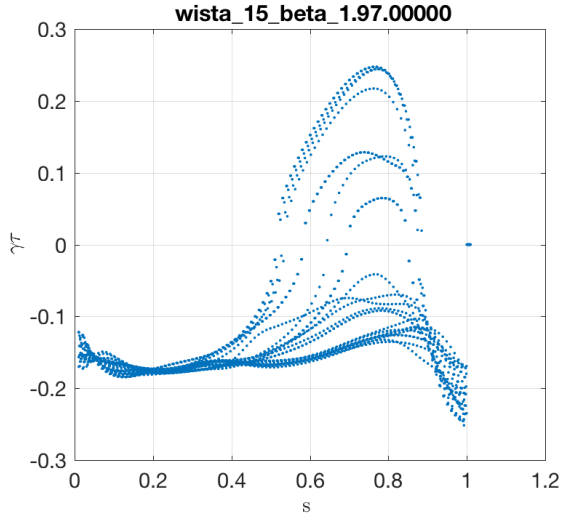
Beta=2.59, OPT_SFINCS_Frombasecase







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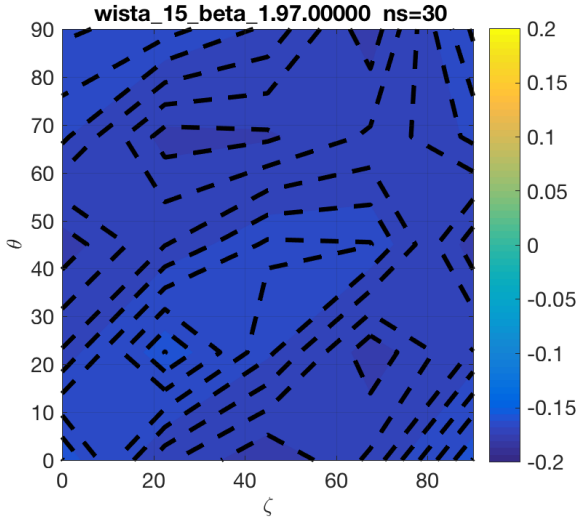
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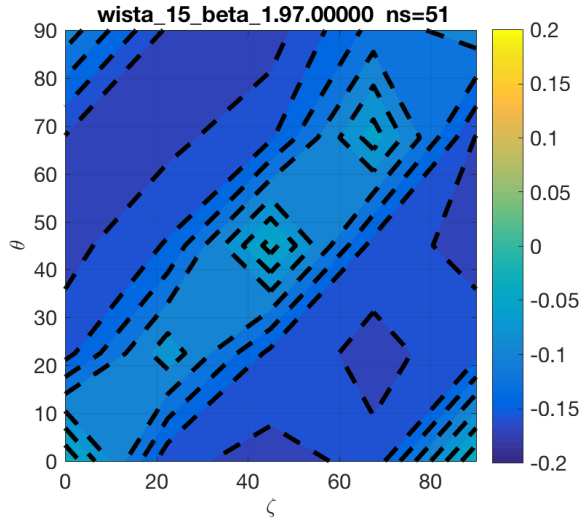
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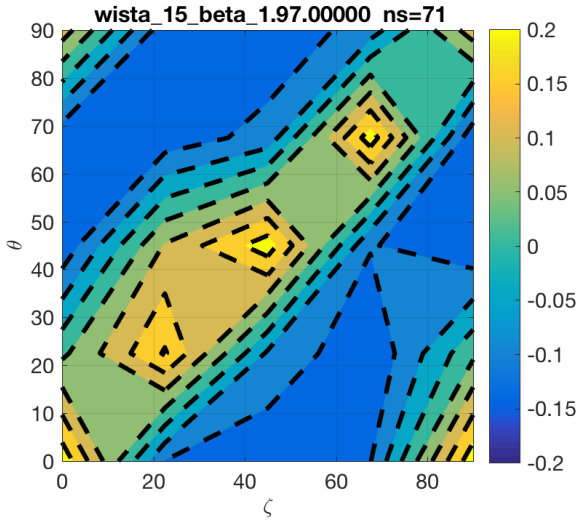
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ζ
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0.1
0.05
0
-0.05
-0.1
-0.15
-0.2

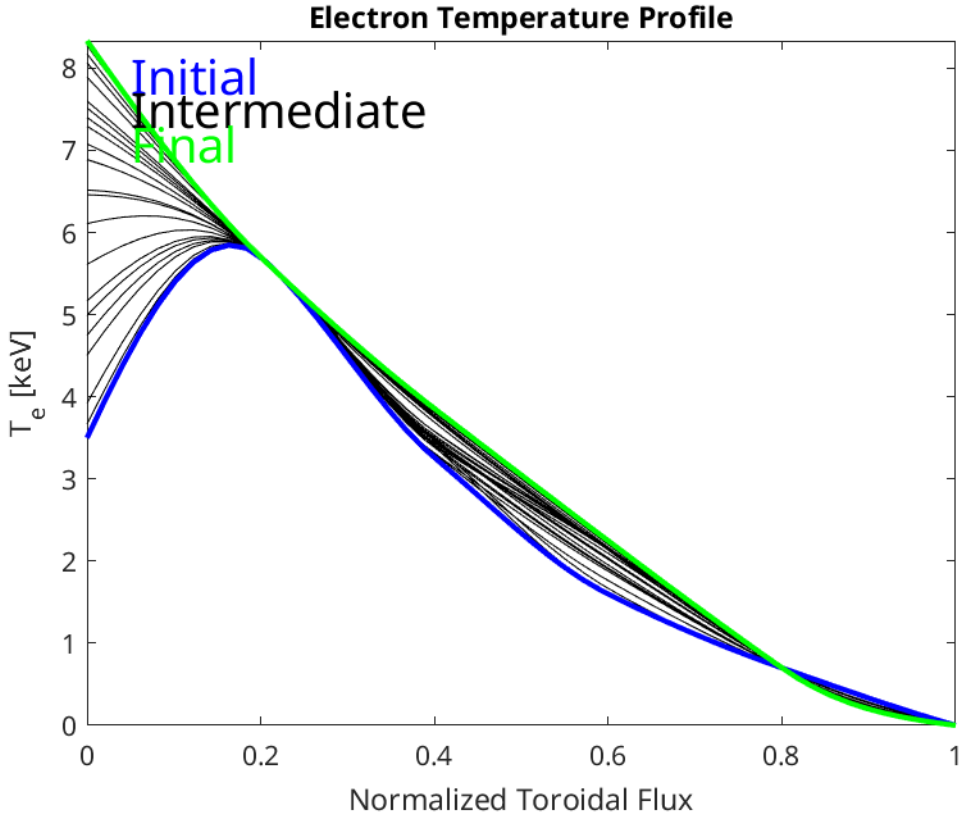
Optimize edge for stability

Targets:

- ballooning at outer ~30% with equal weighting, σ = 1e-3
- Volume avg beta, σ = 1e-4

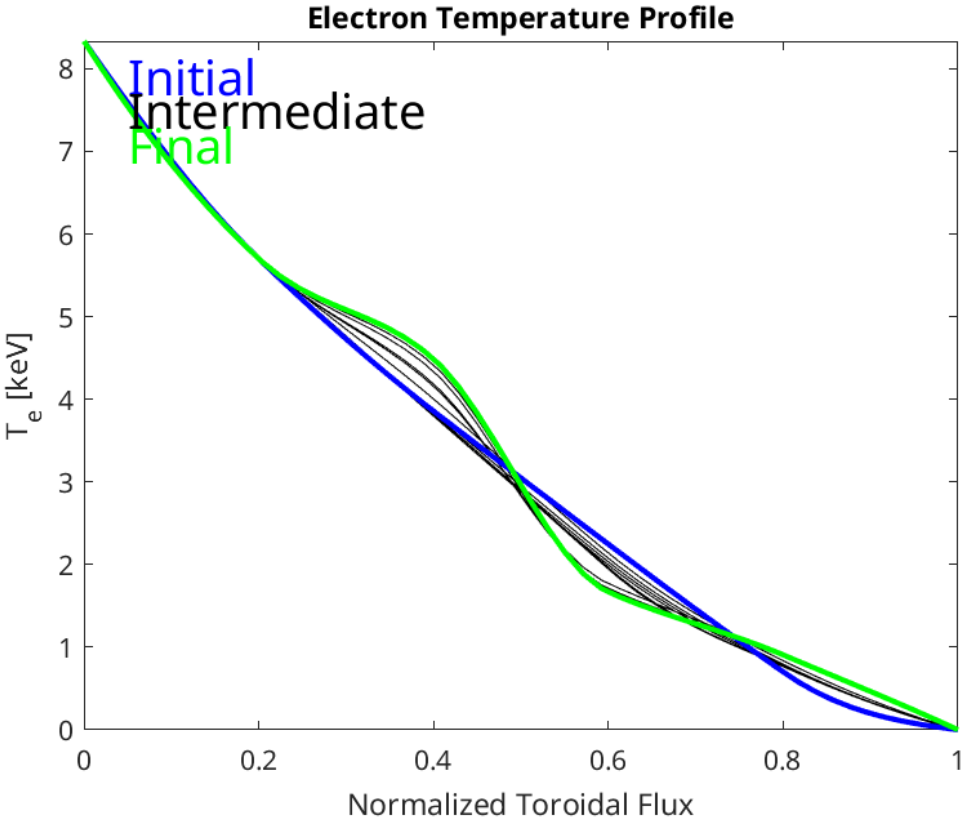
variables:

- TE spline knots (with constraint TI = TE)



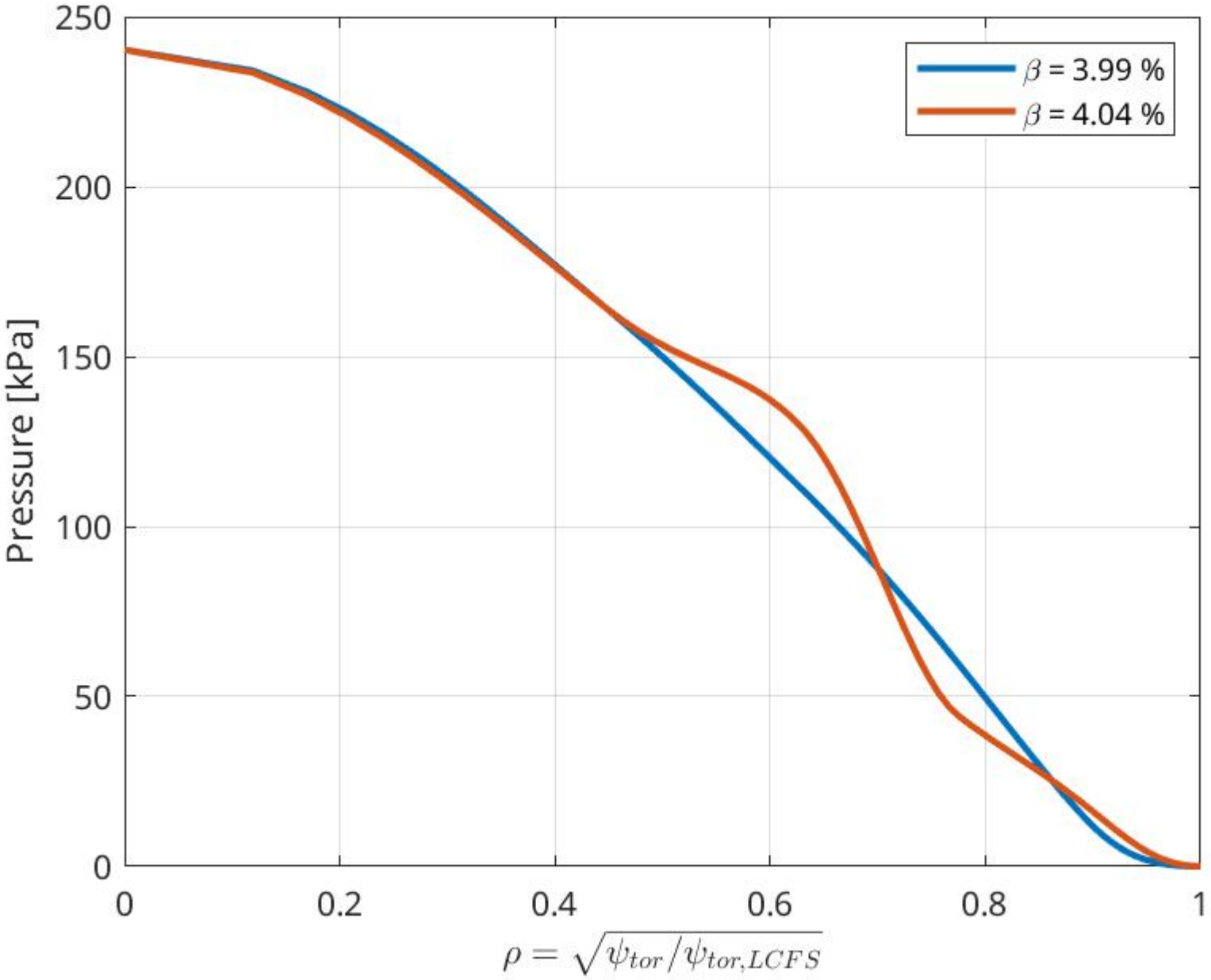
Start from high β solution obtained from optimizing all surfaces for stability

Next vary 0.4, 0.6, 0.8 to stabilize only the outer 30% of flux surfaces



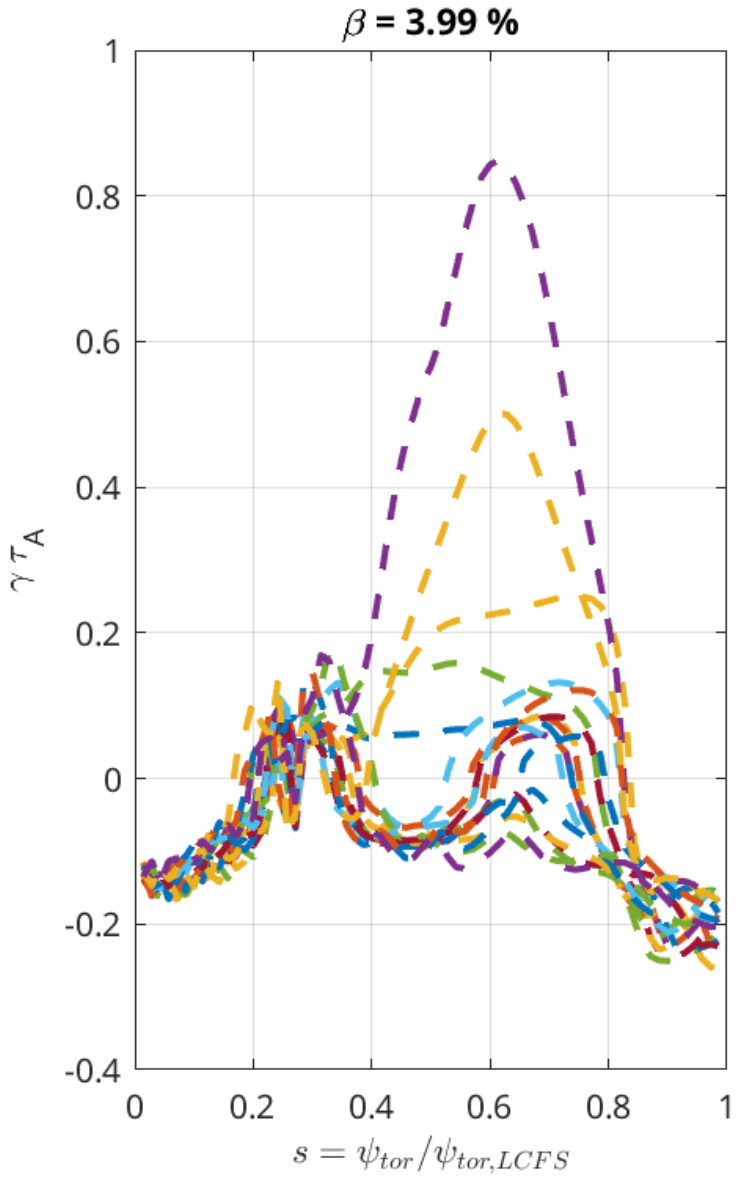
Pressure gradient steepens significantly inside targeted surfaces

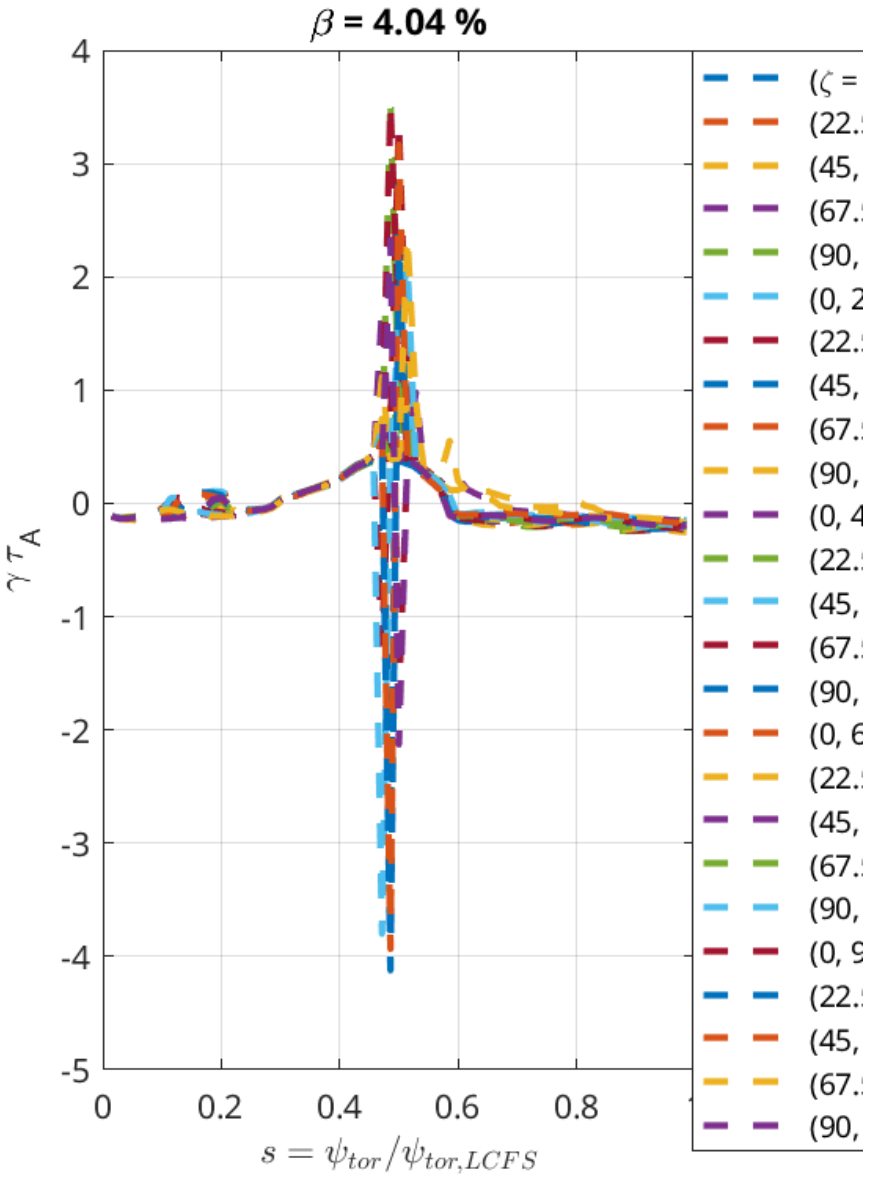
Final result: red curve



Ballooning optimized at angles 0, 22.5, 45, 67.5 and 90

Final result:





Bootsj gives bootstrap current, but significant current on axis – no idea how accurate this current is