Using PEST coordinates and VMEC coordinates:

Cross-code benchmarks of VMEC coordinate transformations and the Γ_c metric

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Outline

- Definitions
  - Cylindrical and PEST coordinates
  - VMEC coordinates
  - Straight field lines coordinates
- Methods
  - Julia and 2 STELLOPT Implementations
- Benchmarks
  - Direct coordinate and B conversions along a magnetic fieldline
  - Derived quantities
  - Surface-averaged metric, Γ_c

Cylindrical and PEST Coordinates

- Cylindrical coordinate system
  - (R, ϕ, Z) is a right-handed orthogonal coordinate system
  - ϕ increases in the counter-clockwise (CCW) direction, when viewed from above
- PEST toroidal coordinate system
  - $(\psi_{PEST}, \Theta_{\text{PEST}}, \zeta_{PEST})$ is a right-handed coordinate system
  - ψ_{PEST} : (Poloidal or toroidal magnetic flux) / 2π
  - Θ_{PEST} : Poloidal angle is a field line label – the same everywhere along the field line.
    - $\nabla\zeta_{PEST} \cdot \nabla\zeta_{PEST} = \frac{1}{R^2}, \nabla\zeta_{PEST} \cdot \nabla\psi = 0, \nabla\zeta_{PEST} \cdot \nabla\Theta = 0$
    - $\zeta_{PEST} = -\phi$
  - ζ_{PEST} increases in the clockwise (CW) direction, when viewed from above

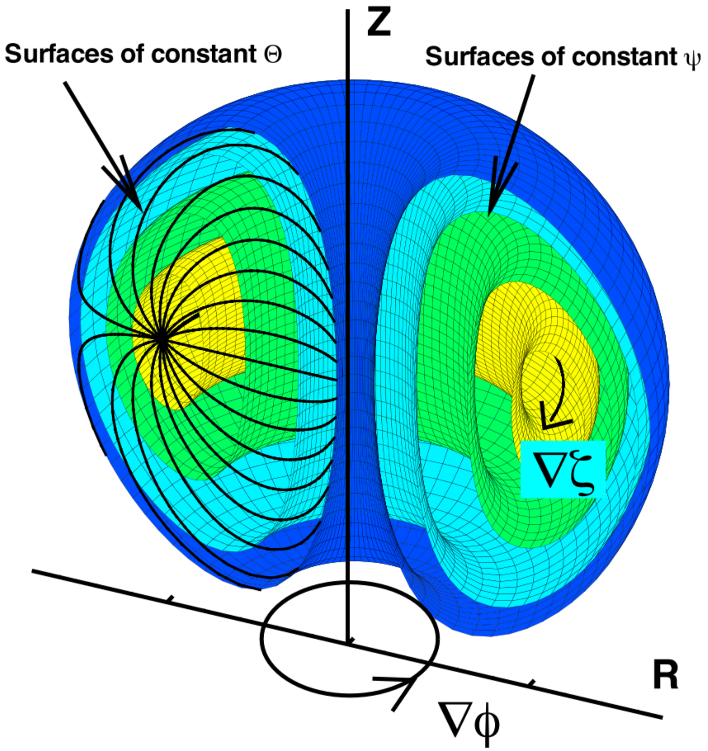


FIG. 1. The definition of the ψ, Θ, ζ flux coordinate system used in this paper illustrated for an extremely low aspect ratio tokamak. The poloidal angle Θ uses a capital theta to distinguish from a cylindrical version of a poloidal angle [$\theta = \tan^{-1}(Z/R)$] as the surfaces are quite different as seen with the symmetric poloidal angle shown (defined later). The toroidal coordinate ζ is in the opposite direction of the R, ϕ, Z cylindrical coordinate system. In general, the planes of constant ζ are not flat as shown here but can vary with ψ and Θ.

S. E. Kruger *and* John M. Greene, Physics of Plasmas **26**, 082506 (2019)

Cylindrical and VMEC Coordinates

- Cylindrical coordinate system is the same
- VMEC Toroidal coordinate system #1
  - $(\psi_{vmec}, \theta_{vmec}, \zeta_{vmec})$ is a left-handed coordinate system
  - ψ_{vmec} : Toroidal magnetic flux $/ -2\pi$
  - $\theta_{VMEC}(\psi_{vmec}, \zeta_{vmec})$: Poloidal angle that maximizes force-balance convergence with minimal spectral content.
  - $\zeta_{vmec} = \phi$

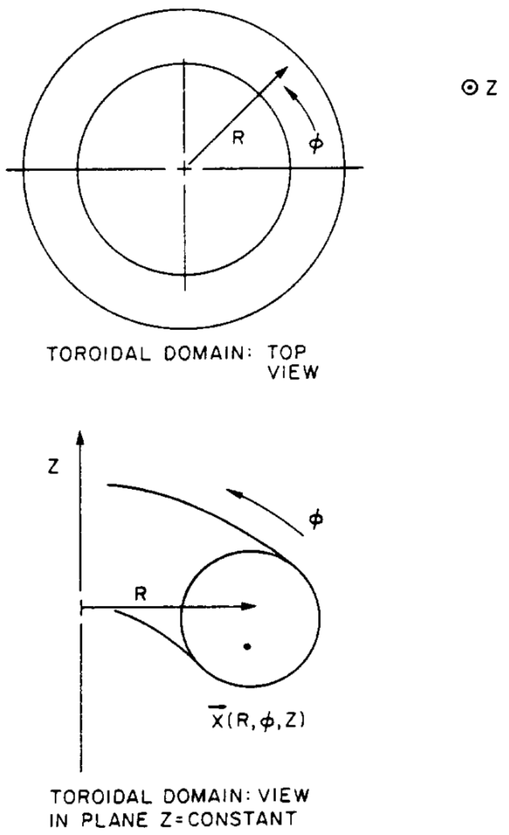


FIG. 1. Toroidal-cylindrical coordinate system.

S. P. Hirshman and J. C. Whitson, The Physics of Fluids 26, 3553 (1983)

VMEC Coordinate System #2

- The internal computational grid of VMEC uses (s,u,v) coordinates, and only spans a single field period
  - $s = \frac{\psi_{VMEC}}{\psi_{VMEC,LCFS}}$: A radial grid coordinate (can be adjusted with 'aphi')
  - $u = \theta_{VMEC}$
  - $v = NFP \times [\phi \ \% \left(\frac{2\pi}{NFP}\right)]$ (%: modulo operator)

Both poloidal angles, θ_{PEST} and θ_{VMEC} increase along the line that starts from the outboard side of the plasma and travels over the the top of the torus as it proceeds towards the inboard side.

Straight field lines coordinates

PEST Coordinates

- $\boldsymbol{B}(\psi_t, \alpha_t, \zeta) = \nabla\psi_t \times \nabla\alpha_t$
- $\alpha = \Theta - \iota_{PEST} * \zeta_{PEST}$
  - The angle that keeps you 'on' the field line Θ

$$\iota = \frac{d\psi_p}{\mathrm{d}\psi_t}$$

Note: Because of the Left-Handed vs. Right-Handed difference between PEST and VMEC:

$$\zeta_{PEST} = -\zeta_{VMEC}$$
$$\iota_{PEST} = -\iota_{VMEC}$$
$$\psi_{t,PEST} = -\psi_{t,VMEC}$$

VMEC Coordinates

- $\boldsymbol{B}(\psi_t, \theta_V^*, \zeta_V) = \nabla\psi_t \times \nabla\theta_V^* + \iota_{\mathrm{VMEC}} \nabla\zeta_V \times \nabla\psi_t$
- $\theta_V^* = \theta_V + \lambda(\psi_t, \theta_V, \zeta_V)$
  - The angle that makes the field lines straight in (θ_V^*, ζ_V)

Converting from a PEST coordinate to VMEC coordinates is the same as finding the root of the following equation:

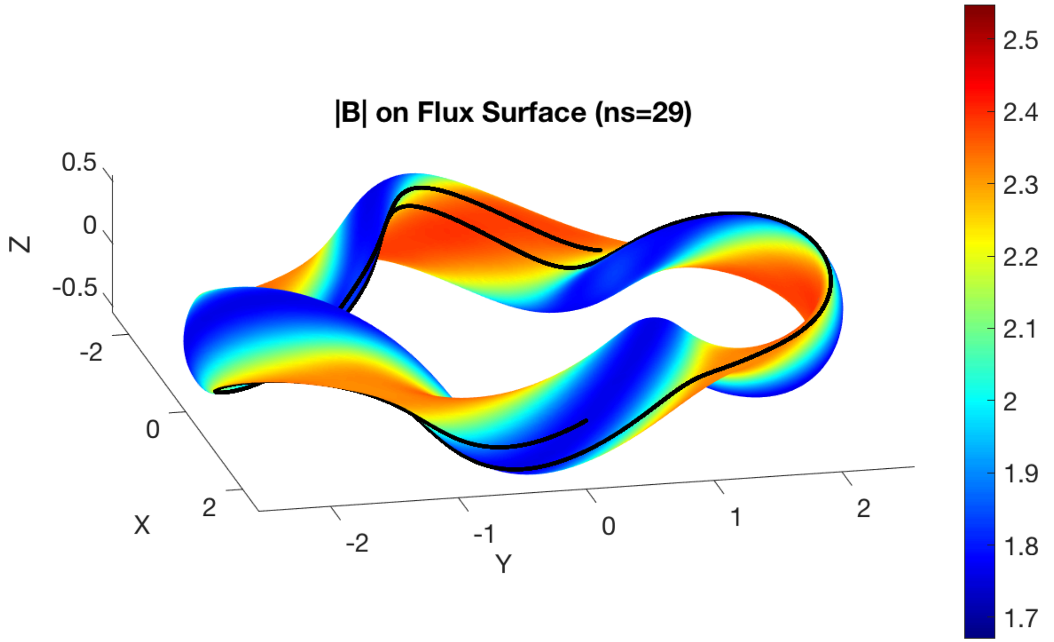
$$\theta_V + \lambda(\psi_t, \theta_V, \zeta_V) - \alpha - \iota_V \zeta_V = 0$$

Methods

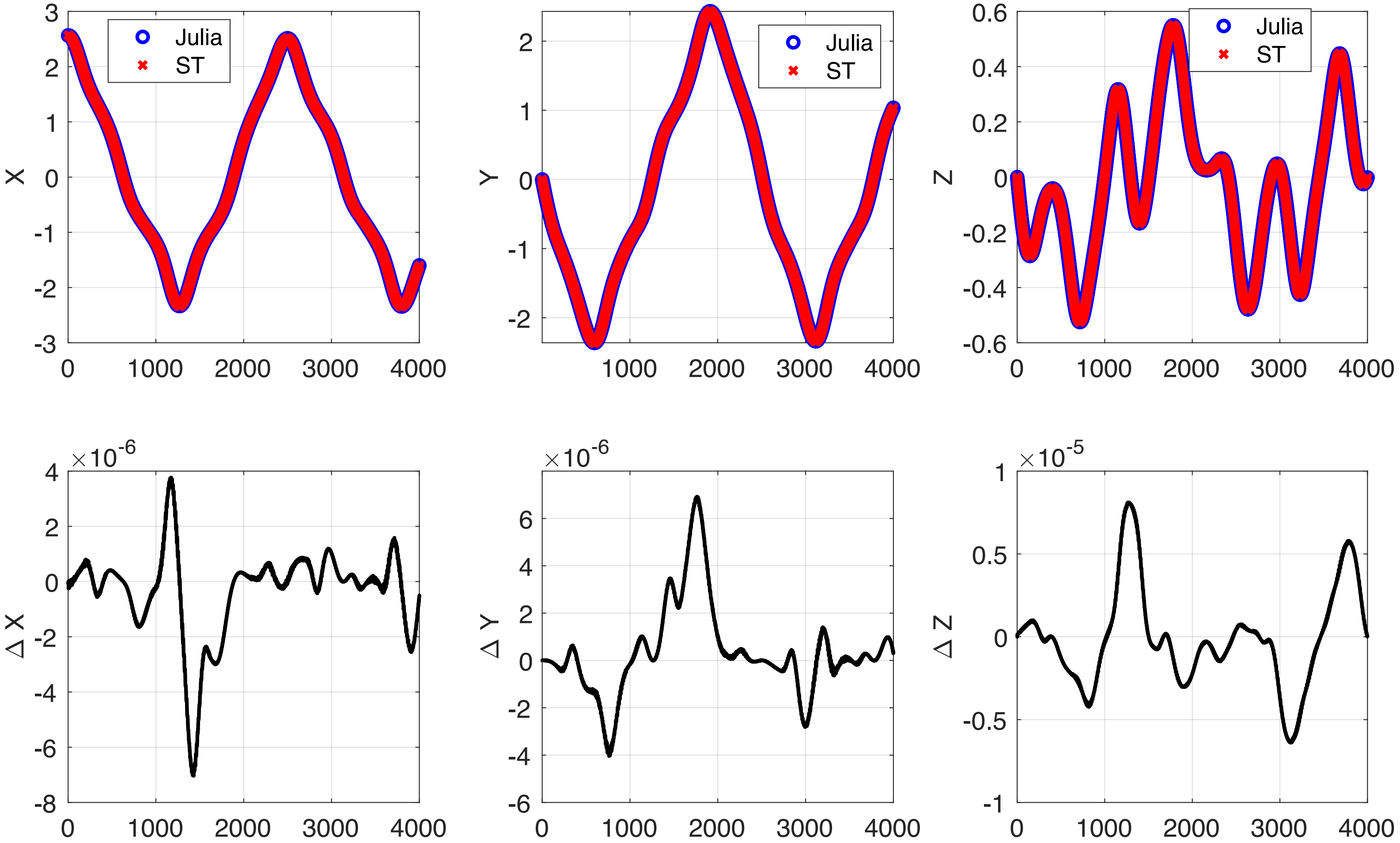
- Julia: See previous WISTELL meetings on PET.jl and VMEC.jl
- STELLOPT
  - https://github.com/PrincetonUniversity/STELLOPT/tree/feature/gamma2
  - LIBSTELL/Modules/stel_tools.f90
  - STELLOPTV2/Sources/Chisq/chisq_gammac.f90
- Also compare to KNOSOS
  - https://github.com/PrincetonUniversity/STELLOPT/tree/CIEMAT

Following a field-line around the torus

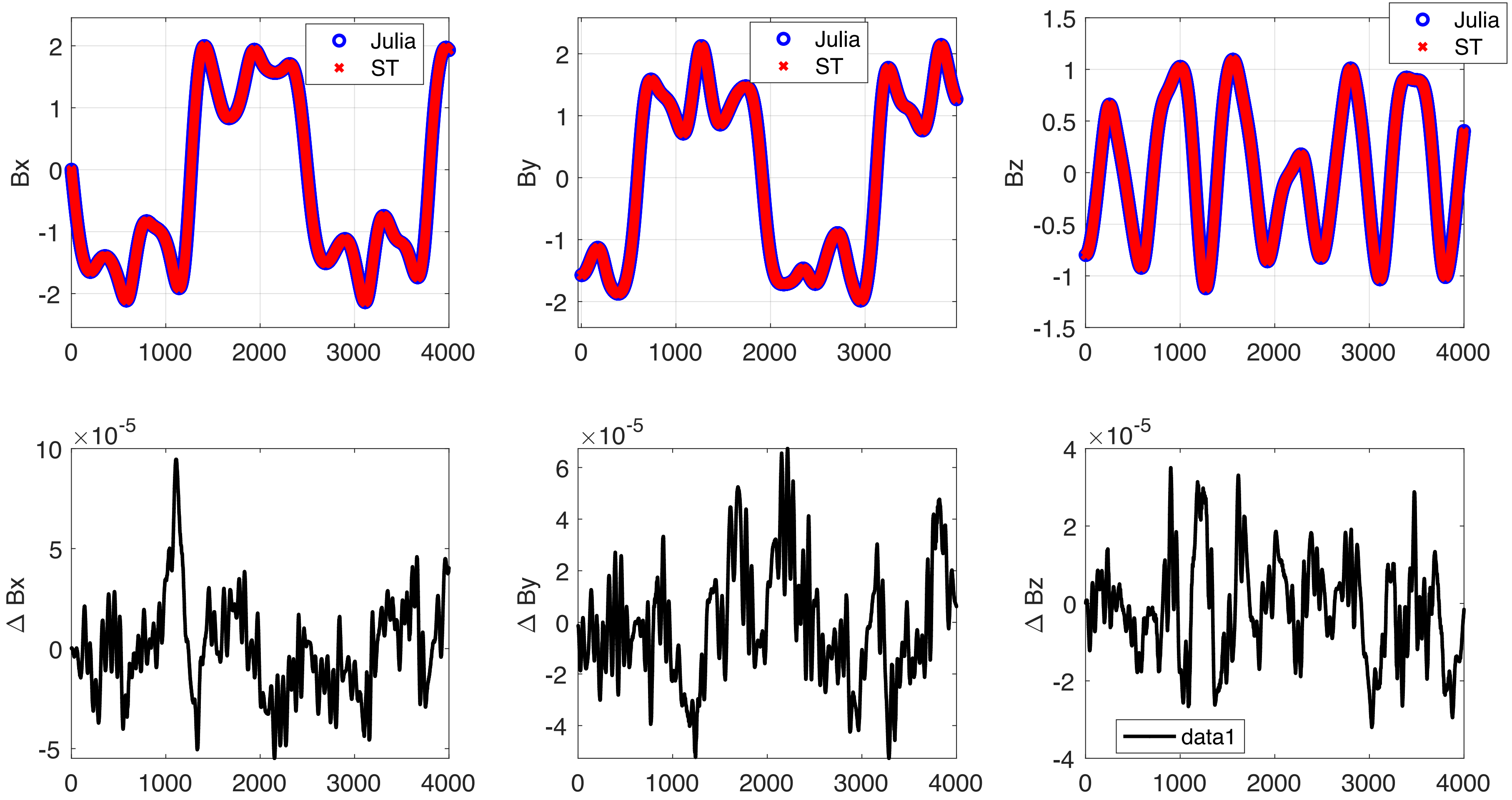
- Calculation of the trajectory of the field line specified by PEST angles
  - |B| on surface 29 of 51 (jet)
  - Points along field line (black points)



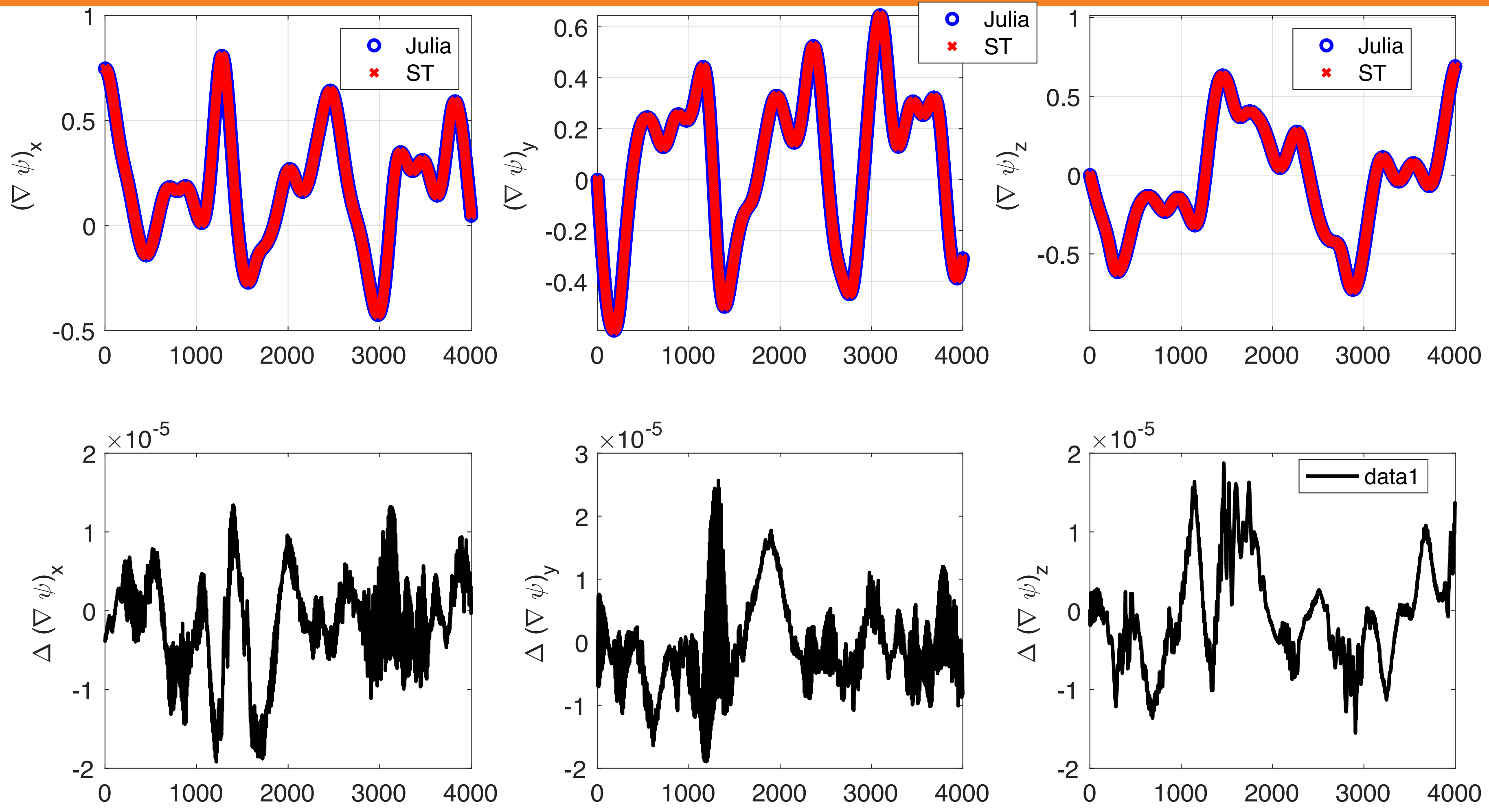
(X,Y,Z) – locations of points along field line



Bx, By and Bz along field line

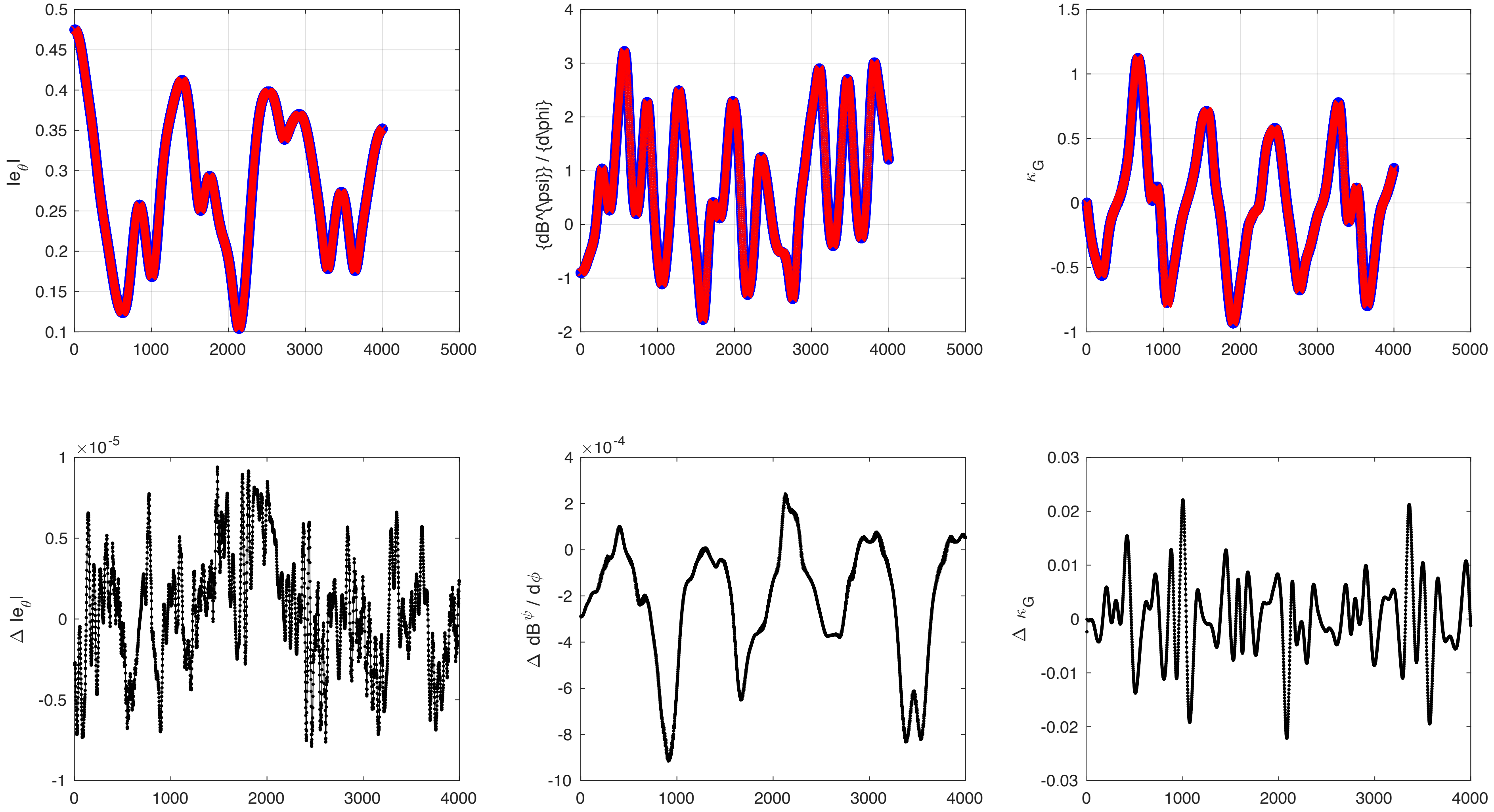


$\nabla\psi$: x-, y-, and z- components along field line



Terms used in the Γ_c metric, along the field line

- $|e_\theta|$
- $\dfrac{dB^\psi}{d\phi}$
- Geodesic curvature: κ_G
  - Factor of -1 still under investigation



Γ_c vs. s in WISTELLA

The remaining sections of the code (well-boundary mapping + integration techniques) are developed separately. Below are comparisons of the final metric calculation across the minor radius for ATEN (WistellA).

Multiple Codes

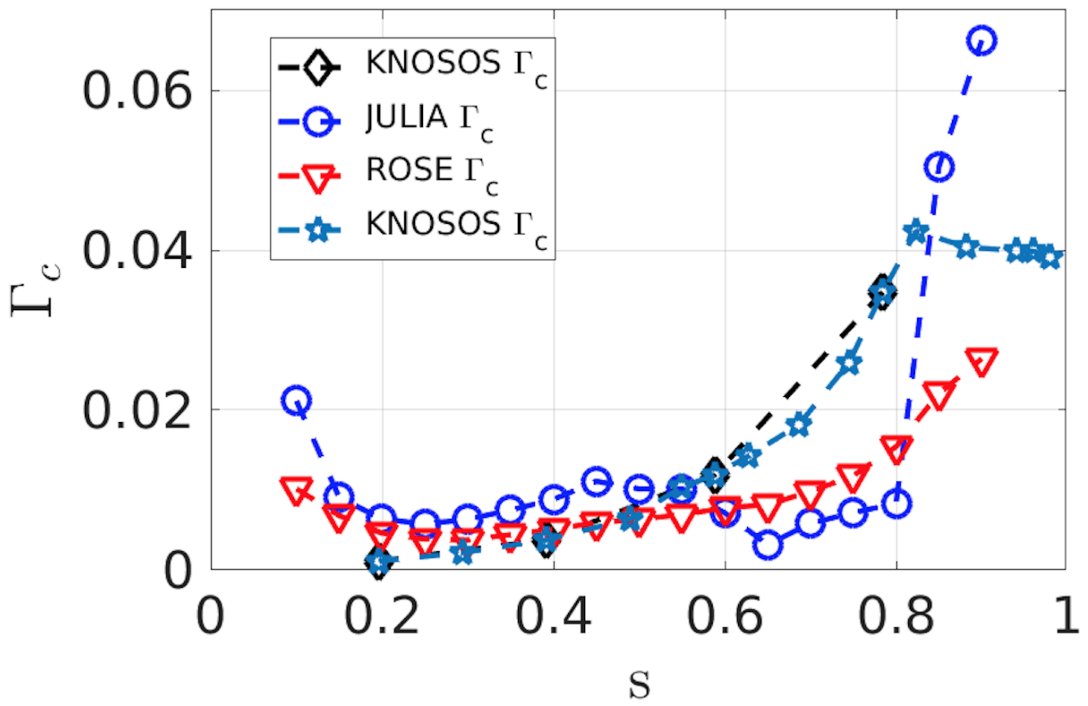
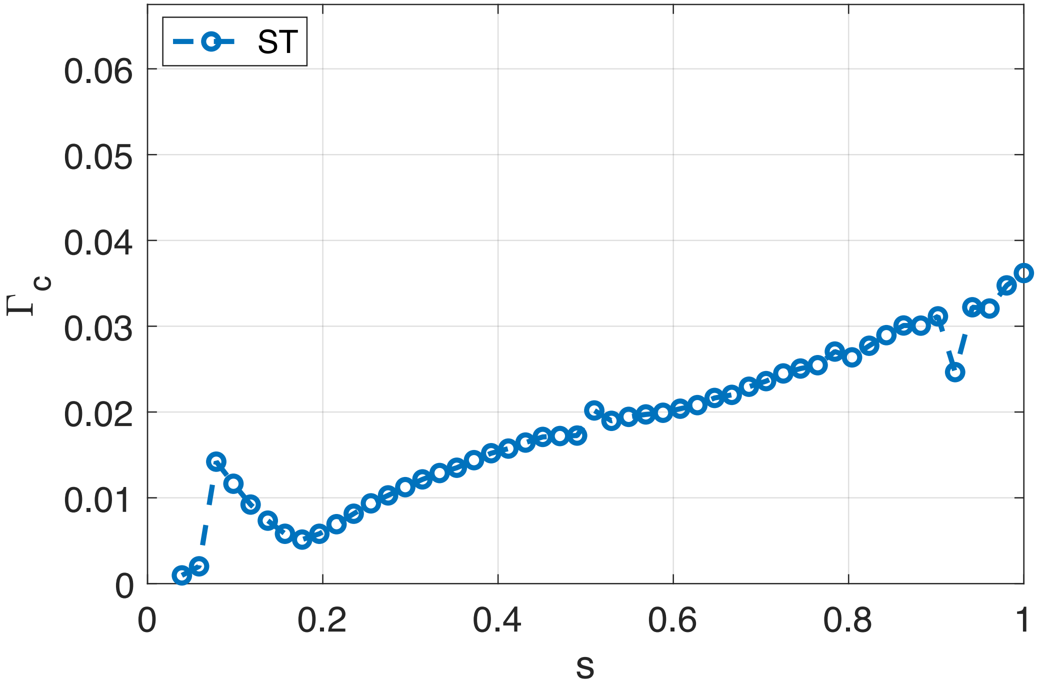


Figure courtesy of E. Sanchez and J.L. Velasco

The most recent 'gammac'



Summary

- Benchmarks show that the mapping technique and subsequent methods are producing similar results
- Other configurations should be checked (QA, QO, non-opt.)
- Convergence testing, etc. not yet completed
  - Results shown for $\Delta\Phi = \frac{1}{400}$ radians and $400/2\pi$ ~ 64 Transits