QS optimization with Lasso

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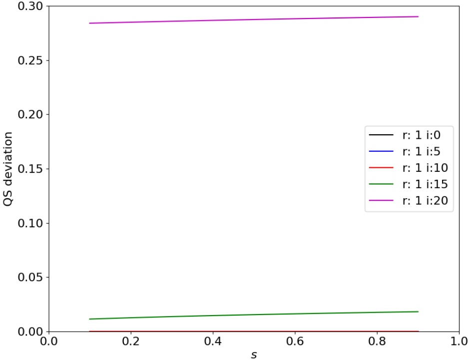
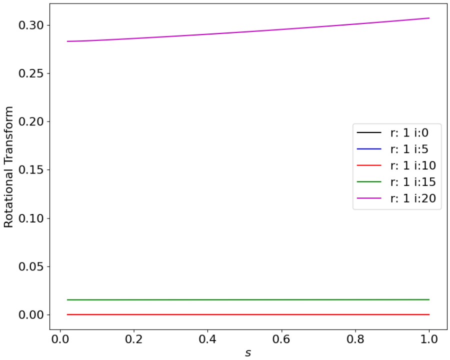
Stellopt Scenarios 2DOF test

- Optimize an equilibrium varying only m=1, n=0 mode
- Target iota at s=0, and QS deviation at s=1
- Due to difficulties of calculation at edge and core, we used iota at s=0.05 and QS deviation at s=0.95

scheme	RBC	ZBS	iota	QS deviation	itertot	1% iter
target	-0.09223	-0.09449	-0.40971	0.17702		
L-BFGS	-0.09185	-0.09440	-0.40972	0.17657	96	23
Conj. grad.	-0.09185	-0.09440	-0.40972	0.17657	51	12

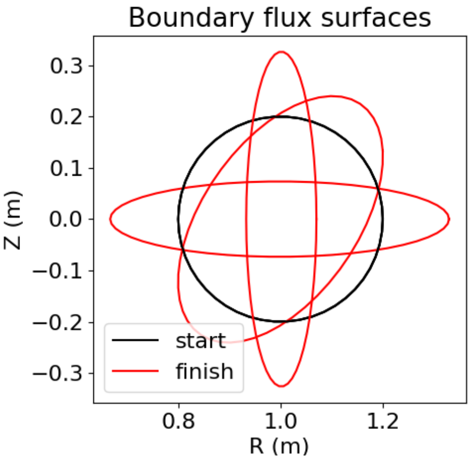
Start from perfectly circular QA



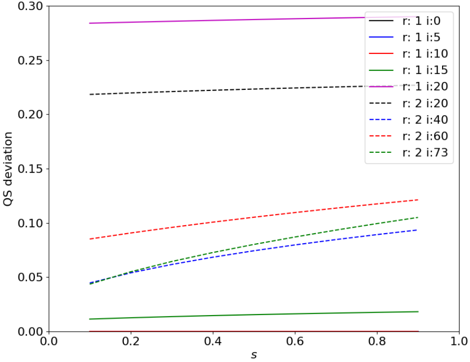


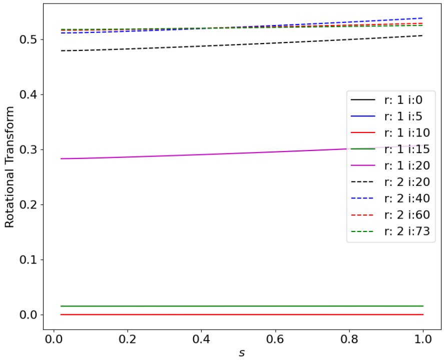
- Only optimize $|m, n| < 1$ modes
- iota weighted 100 times relative to QS
- iota optimized on s=0.1 and 0.7; QS optimized at s = 0.5

Start from perfectly circular QA



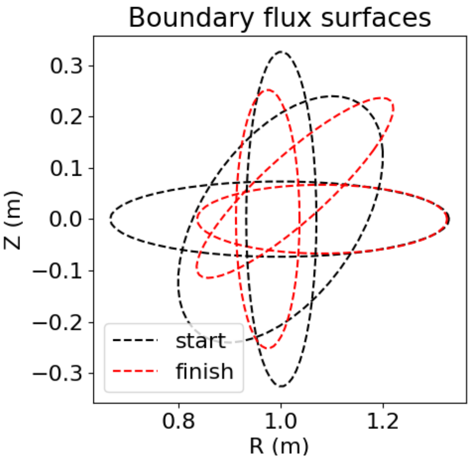
Continue with only using low m, n modes



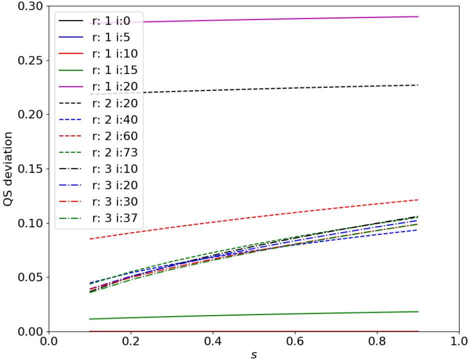


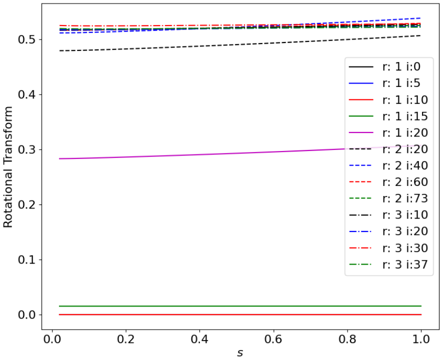
- Only optimize $|m, n| < 1$ modes
- iota weighted 10 times relative to QS
- iota optimized on s=0.1 and 0.7; QS optimized at s = 0.5

Continue with only using low m, n modes



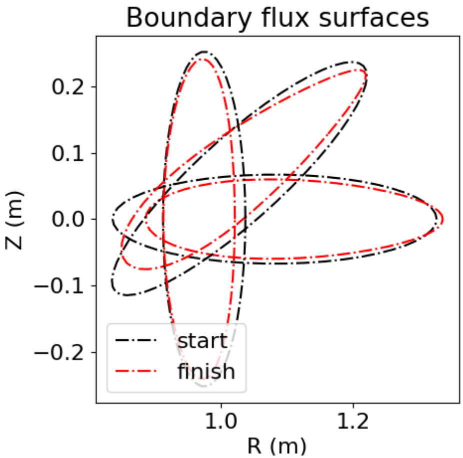
Increase m, n allowable modes



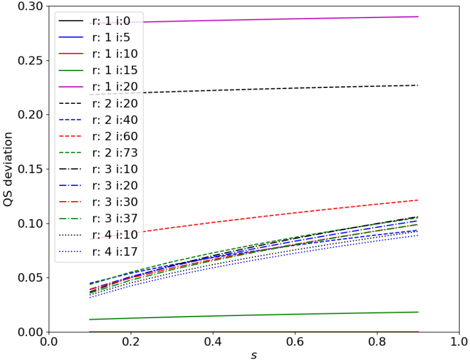


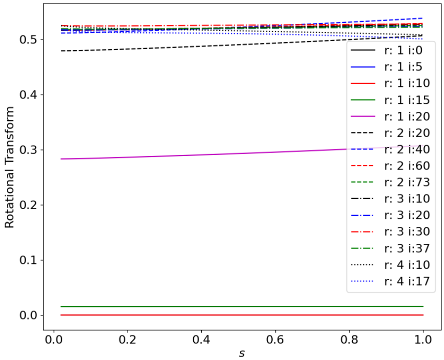
- Only optimize $|m, n| < 2$ modes
- iota weighted 3 times relative to QS
- iota optimized on s=0.1 and 0.7; QS optimized at s = 0.5

Increase m, n allowable modes



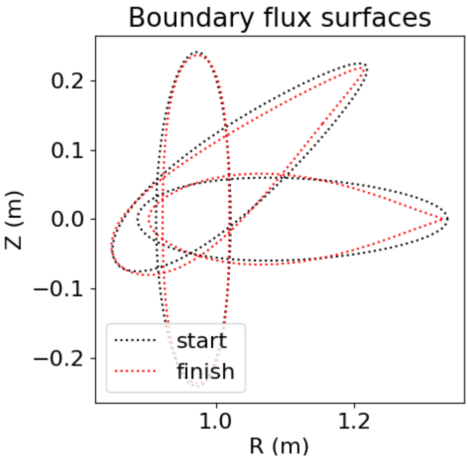
Increase m, n allowable modes to —3— or less





- Only optimize $|m, n| < 3$ modes
- iota weighted 2 times relative to QS
- iota optimized on s=0.1 and 0.7; QS optimized at s = 0.3, 0.5, 0.7

Increase m, n allowable modes to —3— or less



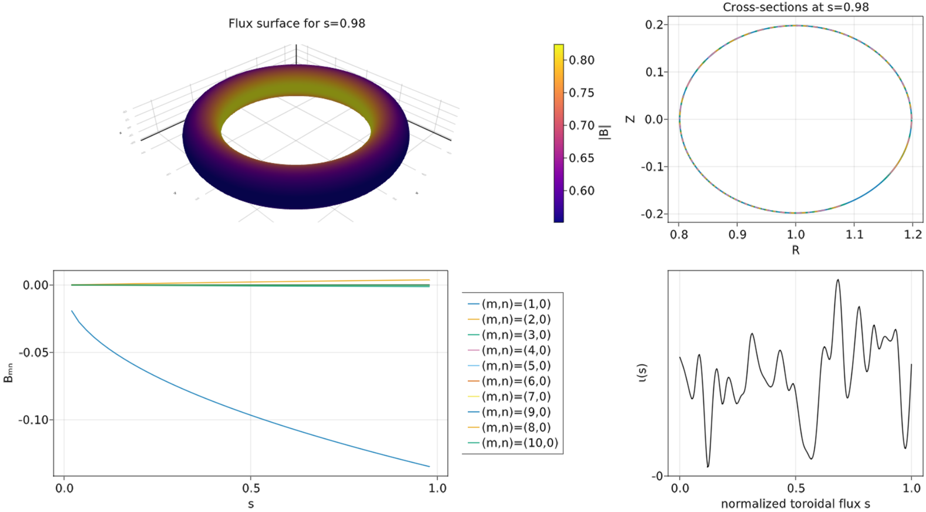
QA Optimization - Take 2

- Attempt to approximate results from Landreman and Paul, 2021
- Objective function:

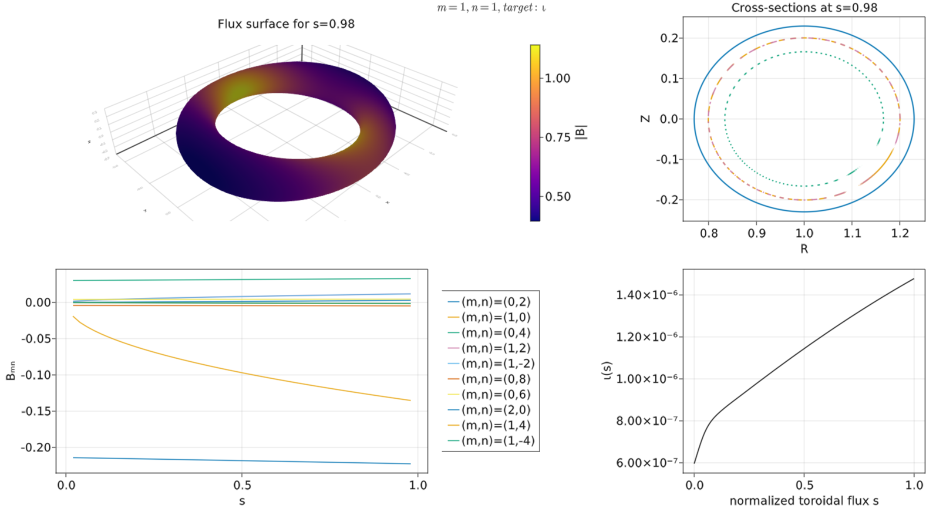
$$f_{QA} = \frac{(A - A_*)^2}{\sigma_A^2} + \sum_{s_i} \frac{f_{qs}^2(s_i)}{\sigma_{qs}^2} + \frac{(\iota(s_i) - \iota_*)^2}{\sigma_\iota^2}, \quad s_i \in [0.1, 0.2, \ldots, 0.9]$$

- $A_* = 5$, $\iota_* = 0.47$, $f_{qs} = \sum_{m,n \neq 1,0} \frac{B_{m,n}^2}{B_{0,0}}$
- Algorithm: Conjugate Gradient method with Hager-Zhang line search
- Recipe:
  1. Start $m = 1, |n| \leq 1$: target ι, A
  2. For $m = 1, |n| \leq 1$: target QS
  3. For $m = 2, |n| \leq 2$: target ι, A
  4. For $m = 2, |n| \leq 2$: target QS
  5. …

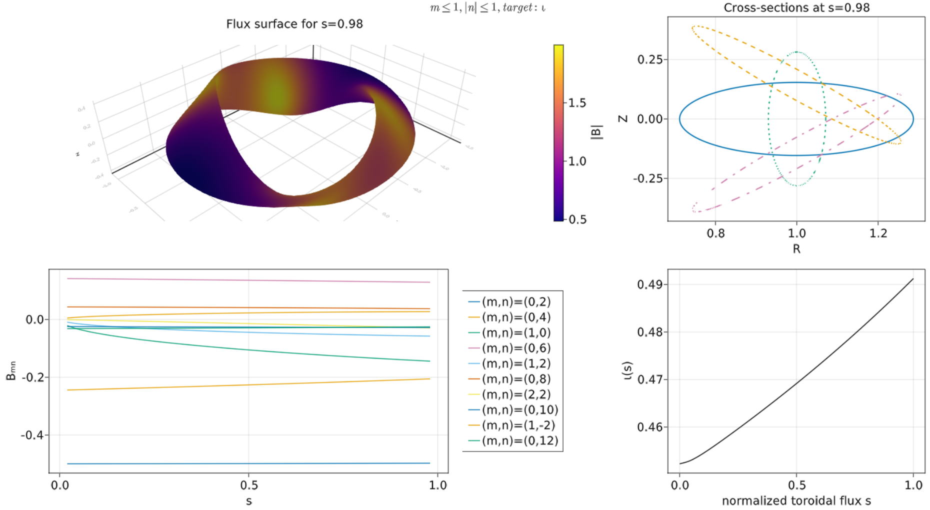
Step 0 - Initial Equilibrium



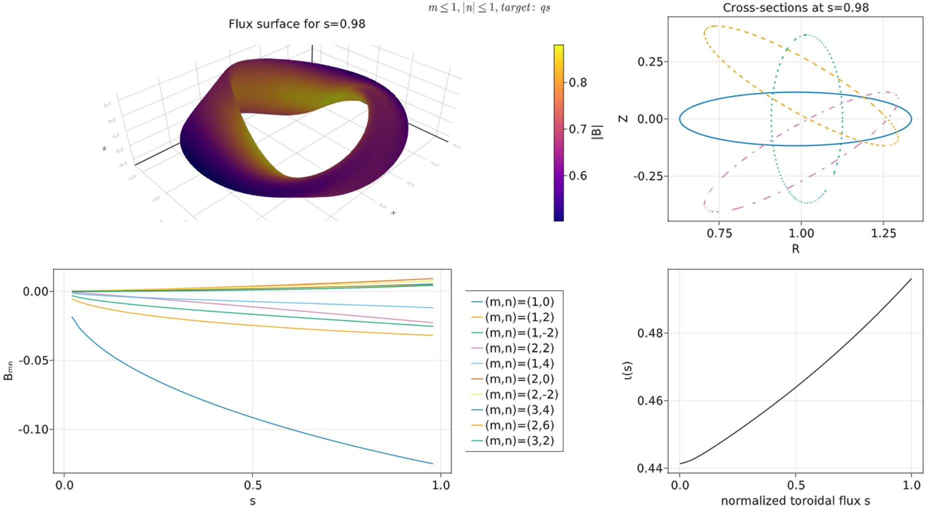
Step 1 - Vary $(m, n) = (1, 1)$ targeting ι



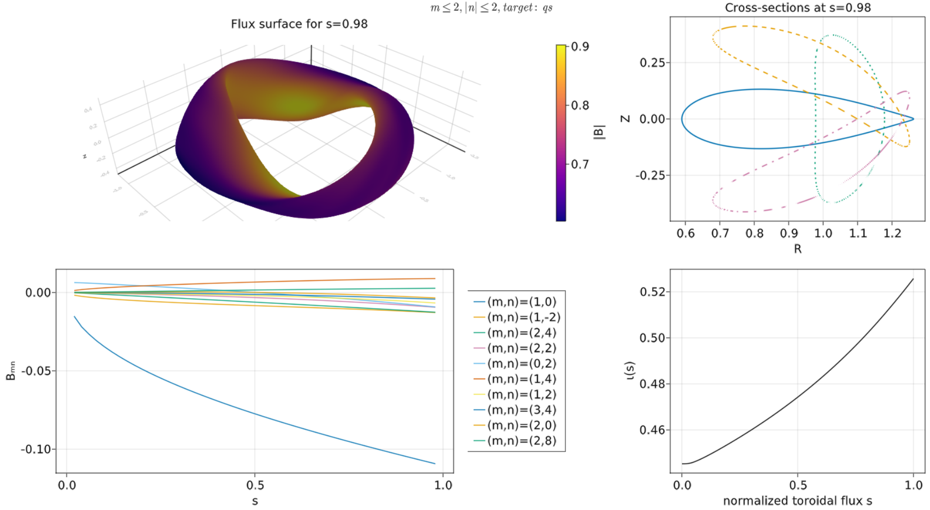
Step 2 - Vary $(m \leq 1, |n| \leq 1)$ targeting ι



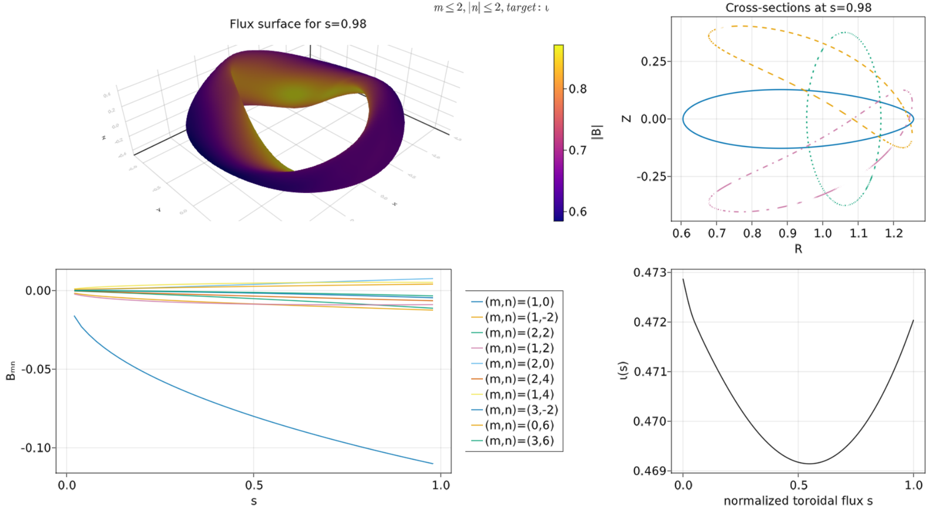
Step 3 - Vary $(m \leq 1, |n| \leq 1)$ targeting QS



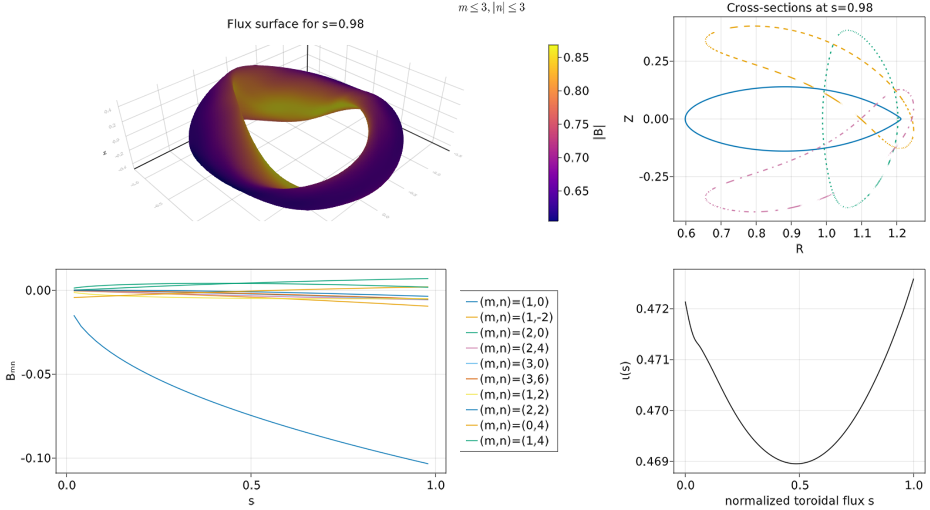
Step 4 - Vary $(m \leq 2, |n| \leq 2)$ targeting QS



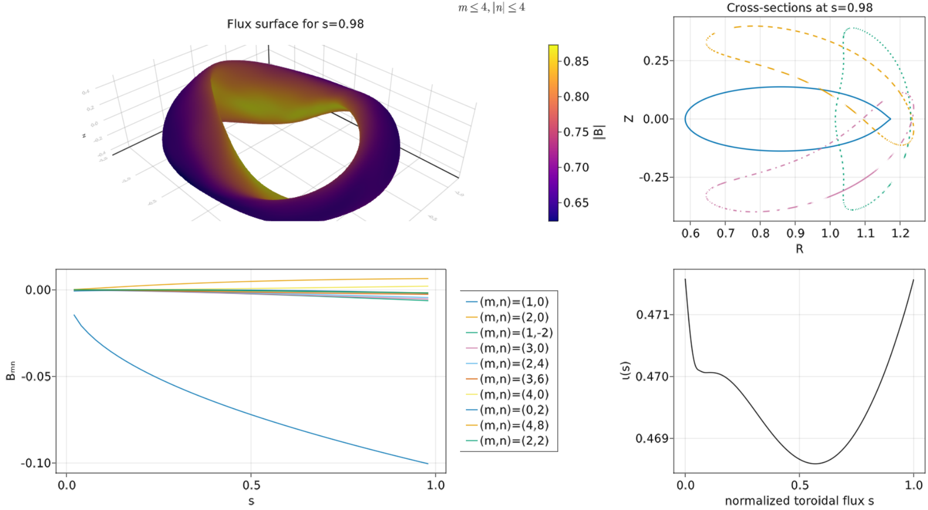
Step 5 - Vary $(m \leq 2, |n| \leq 2)$ targeting ι



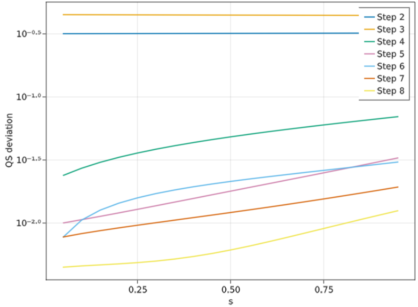
Step 6 - Vary $(m \leq 3, |n| \leq 3)$ targeting QS

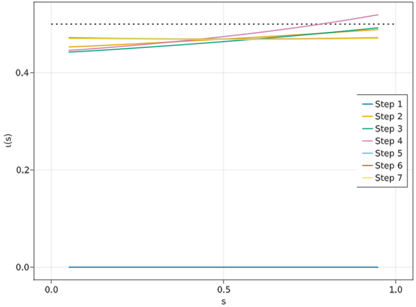


Step 7 - Vary $(m \leq 4, |n| \leq 4)$ targeting QS



QA Summary





- Quasisymmetry below NCSX, below IPP-QA for most s
- None of the steps reached convergence, only terminated due to step/time limits $\Rightarrow$ more room for optimization

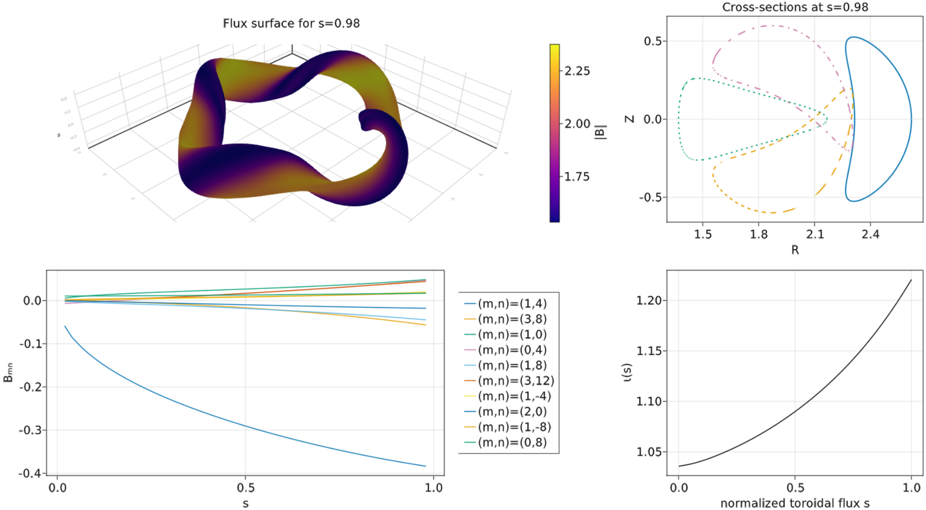
QH Optimization

- Attempt to replicate/improve Aaron's previous result
- Objective function:

$$f_{QA} = \frac{(A - A_*)^2}{\sigma_A^2} + \sum_{s_i} \frac{f_{qs}^2(s_i)}{\sigma_{qs}^2} + \frac{(\iota(s_i) - \iota_*)^2}{\sigma_\iota^2}, \quad s_i \in [0.14, 0.28, 0.42, 0.56, 0.70, 0.84]$$

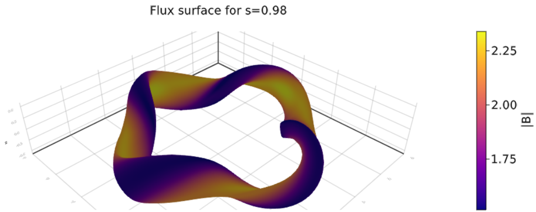
- $A_* = 6.697$, $\iota_* = 1.07$, $f_{qs} = \sum_{m,n \neq 1,0} \frac{B_{m,n}^2}{B_{0,0}}$
- Algorithm: Conjugate Gradient method with Hager-Zhang line search
- Recipe:
  1. Start $m \leq 3, |n| \leq 3$: target QS, A
  2. Increase to $m \leq 4, |n| \leq 4$: target QS, A
  3. Target ι, QS, A
  4. ...

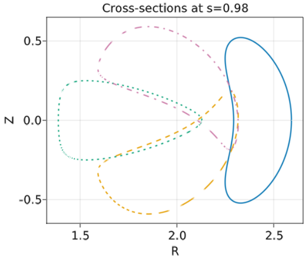
Step 0 - QHS 46

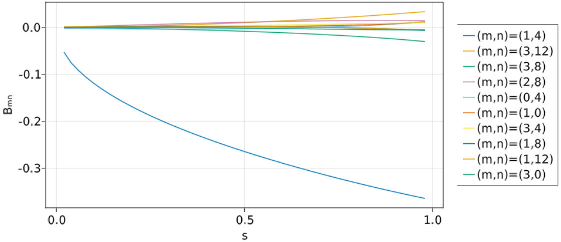


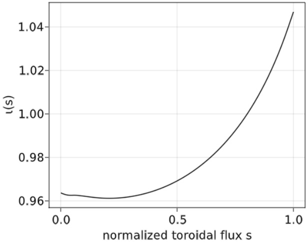
Flux surface for s=0.98
2.25
2.00
1.75
|B|
Cross-sections at s=0.98
0.5
0.0
-0.5
Z
1.5
1.8
2.1
2.4
R
0.0
-0.1
-0.2
-0.3
-0.4
Bmn
0.0
0.5
1.0
s
(m,n)=(1,4)
(m,n)=(3,8)
(m,n)=(1,0)
(m,n)=(0,4)
(m,n)=(1,8)
(m,n)=(3,12)
(m,n)=(1,-4)
(m,n)=(2,0)
(m,n)=(1,-8)
(m,n)=(0,8)
1.20
1.15
1.10
1.05
ι(s)
0.0
0.5
1.0
normalized toroidal flux s

Step 1 - Target QS

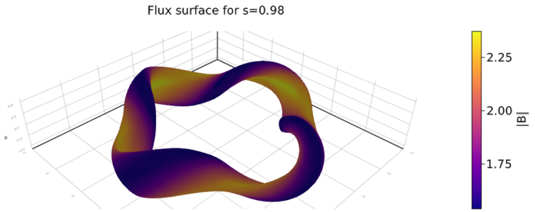


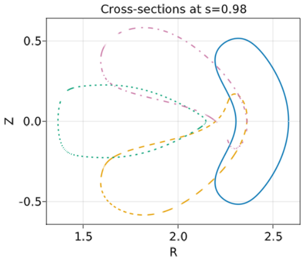


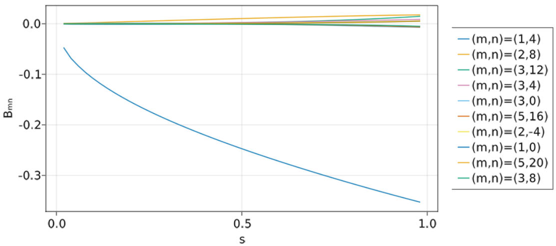


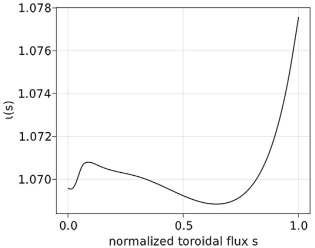


Step 2 - Target QS and ι









Conclusions

- Currently QS optimization not yet as good as Landreman and Paul results
  - Many optimizations did not reach converged state, a lot of room for improvement
- Enforcing only QS and ι seems to lead to strong shapes, need flux surface curvature constraints
  - High-dimensional space, with constraints there is still likely a path to better QS and ι
- Better ways to optimize, i.e. Conjugate Gradient vs. trust-region reflective? Does the formulation of the QS metric matter?